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Synthesis of Lossy Reflection-Mode Bandstop Filters

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***Abstract* — Synthesis of lossy reflection-mode bandstop filters is discussed. Equations, data, and algorithms for the transmission characteristics and circuit element values of maximally-flat and equiripple absorptive bandstop filters are given and examples of their use are provided.**

I. Introduction

It has been well known, since at least the reflection-type phase shifter [1] work of the early 1960's, that the reflection characteristic of a one-port network ***S*** can be converted into the transmission characteristic of a two-port by using either a circulator or a 3-dB hybrid or directional coupler, as shown in Fig. 1. Filters employing this concept are referred to as "non-reflective filters" [2], "reflection filters" [3], reflection networks" [4], "reflection-mode filters" [5], "absorptive filters" [6]-[8], and "perfectly-matched filters" [9]. This paper generalizes and extends Rhodes's theory of lossy bandstop filters of this type [5] in order to aid in the synthesis of lossy filters with maximally flat passband and/or equiripple stopband characteristics.

II. Lossy Reflection-Mode Bandstop Filters

Assuming equal source and load admittances Y_s and an ideal circulator as in Fig. 1(a) or an ideal 3-dB hybrid coupler as in Fig. 1(b) (each with port admittances Y_s), the transmission characteristic of the two-port reflection-mode filter $\boldsymbol{S_t}$ is simply the reflection characteristic of the one-port network ***S***, which is a function of the input admittance Y_{in} (s) of ***S***, where $s=j\omega$ is the complex frequency variable (assuming a sinusoidal input signal):

$$S_{t,21}(s) = S_{11}(s) = (Y_s - Y_{in}(s))/(Y_s + Y_{in}(s)) \cdot \qquad (1)$$

A variety of other reflection-mode-network topologies can be derived by applying circuit transformations to the networks of Fig. 1. For example, portions of networks ***S*** can be absorbed [10] into a hybrid coupler, resulting in truncated one-port networks ***S´***, as in Fig. 2 [7]-[9].

While ***S*** could be any network, for simplicity let ***S*** be a lossy ladder network of admittance inverters J_{r-1} coupling shunt parallel-resonant admittances Y_r with unloaded Qs $Q_r=2\pi f_o C_r/g_r$ and resonant frequency $f_o=(L_r C_r)^{-1/2}$ as in Fig. 3(a), for integer r=1 to n. The corresponding "highpass reflection" prototype of inverter coupled lossy shunt admittances $y_r=g_r+sc_r$ with unloaded Qs $q_r=\omega_h c_r/g_r$ at radian frequency $\omega=\omega_h$ is in Fig. 3(b).

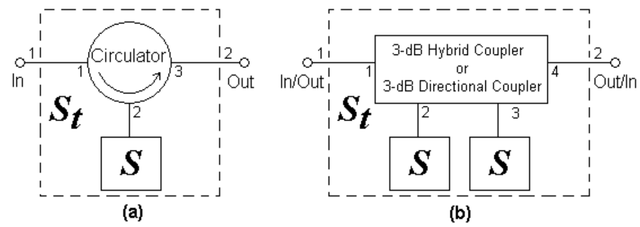

Fig. 1. Reflection-mode filter networks based on (a) a circulator and on (b) a 3-dB hybrid coupler or directional coupler [5],[14].

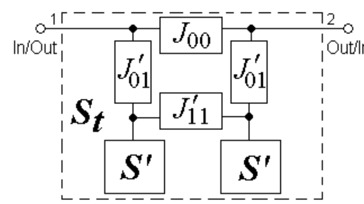

Fig. 2. A reflection-mode filter network in which the first admittance inverters of one-port networks ***S*** have been absorbed into the 3-dB hybrid coupler of Fig. 1(b) resulting in truncated one-port networks ***S'*** and modified admittance inverters J_{01}' and J_{11}'. For one-port networks ***S*** as in Fig. 3, ***S'*** is identical to ***S*** without its first inverter J_0, and the modified inverters become $J'_{01} = \sqrt{2}\, J_0$ *and* $J'_{11} = J_0^2/Y_s$ with $J_{00}=Y_s$.

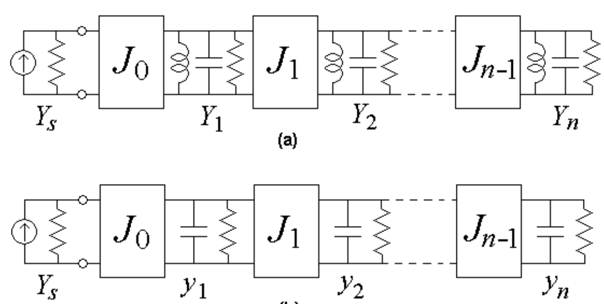

Fig. 3. One-port ladder networks with (a) "bandstop" and (b) "highpass" reflection characteristics.

A. Maximally-Flat Reflection-Mode Bandstop Filters

The generic "maximally-flat reflection-passband" characteristic of the prototype of Fig. 3(b) is given by:

$$S_{11}(s) = \pm s^n \prod_{r=1}^{n} \left(s - j\sigma_o\, e^{j\theta_r} \right)^{-1} \qquad (2)$$
$$= \pm s^n (s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0)^{-1}$$

where the left-half-plane (LHP) poles of (2) appear at

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14. ABSTRACT
Synthesis of lossy reflection-mode bandstop filters is discussed. Equations, data, and algorithms for the transmission characteristics and circuit element values of maximally-flat and equiripple absorptive bandstop filters are given and examples of their use are provided.

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$$s_r = j\sigma_o e^{j\theta_r} = \sigma_o(-\sin\theta_r + j\cos\theta_r) \quad \text{for} \quad r = 1\ldots n \ . \tag{3}$$

These poles lie on a circle of radius σ_o centered at the origin in the complex s plane (i.e., σ_o scales the filter bandwidth). The constant angular separation θ between the poles varies from zero for the equal-resonator-Q_u filter to π/n for the ideal lossless filter. Between these extremes of θ lays the continuum of possible maximally flat characteristics for unequal-resonator-Q_u reflection filters, examples of which are shown in Table I and graphically illustrated in Fig. 4. In general,

$$\theta_r = \theta\,\frac{(2r + x - (n+1))}{2}, \quad \theta = \frac{\pi}{x}, \quad \text{and } x \ge n. \tag{4}$$

These filters may be synthesized by forming the input admittance Y_{in} of the one-port ladder network

$$Y_{in} = \frac{1 - S_{11}}{1 + S_{11}} = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0}{2s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0} \tag{5}$$

and sequentially extracting the element values from it by means of a continued fraction expansion as in [11]. Using (2), (4), and Linnebach's technique, as explained in [12], the coefficients a of (2) and (5) are found to be

$$a_0 = \sigma_o^n \text{ and } a_r = \frac{a_{n-r}}{\sigma_o^{2r-n}} = \sigma_o^{n-r}\prod_{\mu=1}^{r}\frac{\cos\left(\frac{\pi}{2x}\left(x + \mu - (n+1)\right)\right)}{\sin\left(\frac{\pi}{2x}\mu\right)}. \tag{6}$$

By extracting the first capacitance and conductance, c_1 and g_1, from the input admittance of the one-port ladder network as given by (5) and (6), the maximum required unloaded Q of the shunt admittances is found to be

$$q_{\max} = q_1 = \frac{\omega_h c_1}{g_1} = \frac{2\omega_h}{a_{n-1} - 2(a_{n-2}/a_{n-1})} = \left(\frac{2\omega_h}{\sigma_o}\right)\cos\left(\frac{\pi}{2x}\right)\Big/\cos\left(\frac{\pi}{2x}n\right), \tag{7}$$

where ω_h is the radian frequency at which the q is defined. In the conventional case, $x=n$ and $q_{\max}=\infty$; in the equal-Q_u case, $x=\infty$ and $q=2\omega_h/\sigma_o$; and in the case described in [5], $x=n+1$ and $q_{max}=(2\omega_h/\sigma_o)\tan((\pi/2)(n/(n+1)))$.

The squared magnitude of the generic maximally-flat reflection prototype response (2) is given by

$$|S_{11}(\omega)|^2 = \omega^{2n}\Big/\left(\omega^{2n} + \sigma_o^{2n} + \sum_{r=1}^{n-1}\sigma_o^{2(n-r)}b_r\,\omega^{2r}\right),\ b_r = b_{n-r} \ge 0, \tag{8}$$

where the b_r are constants and for which the first $2n$-1 derivatives with respect to ω vanish as ω approaches 0 and ∞. In the lossless case ($q_r=\infty$) $b_r=0$ and in Rhodes's case [5] $b_r=1$. When the resonators have the same values of g and Q_u, (8) becomes

$$|S_{11}(\omega)|^2 = \left(\frac{\omega^2}{\omega^2 + \sigma_o^2}\right)^n = \omega^{2n}\Big/\left(\sum_{r=0}^{n}\binom{n}{r}\sigma_o^{2(n-r)}\,\omega^{2r}\right), \tag{9}$$

where $\sigma_o = 2\ \omega_h / q = 2\ g / c$, the LHP poles of (2) have collapsed to the point $s_r=-\sigma_o$, and the inverter values are

$$J_0 = \pm\sqrt{n\,g\,Y_s} \tag{10}$$

$$J_r = \pm g\sqrt{\frac{(n-r)(n+r)}{(2r-1)(2r+1)}} \text{ for } r = 1\ldots(n-1), \tag{11}$$

x	θ_r	θ	Description
∞	$\pi/2$	0	equal resonator Q_u
$2n$	$\pi(2r+(n-1))/(4n)$	$\pi/(2n)$	small range in Q_u
$n+1$	$\pi r/(n+1)$	$\pi/(n+1)$	Rhodes' choice [5]
$n+½$	$\pi(4r-1)/(2(2n+1))$	$2\pi/(2n+1)$	large range in Q_u
n	$\pi(2r-1)/(2n)$	π/n	infinite Q_u

Table I. Relations for x, θ_r, and θ for maximally-flat prototype characteristics with progressively greater selectivity and greater variation in successive corresponding resonator Q_u.

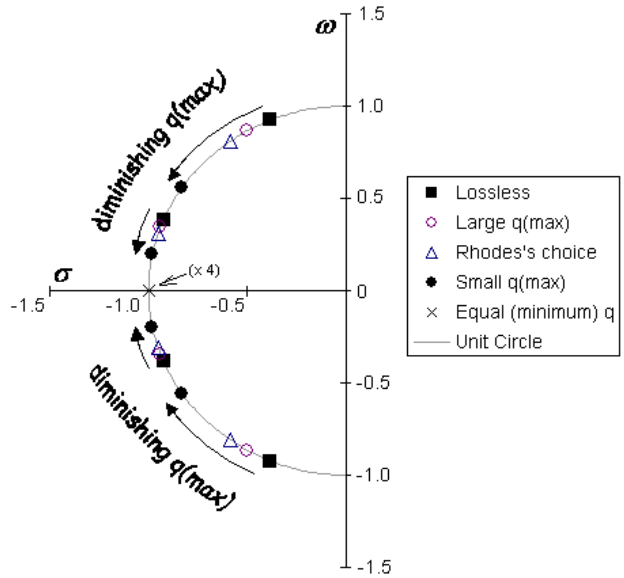

Fig. 4. Prototype LHP maximally-flat reflection poles for n=4, plotted in the complex $s=\sigma+j\omega$ plane, for five different sets of shunt admittance unloaded Qs, showing the trend in the four pole locations as the maximum q diminishes.

For reflection poles on the unit circle, pick σ_o=1 or $q=\omega_h$; for 3-dB return loss at ω_h=1, choose $\sigma_o = (2^{1/n} - 1)^{1/2}$ or

$$q = 2\,\omega_h/\sigma_o = \omega_h c/g = 2\Big/\sqrt{2^{1/n} - 1}\text{; and} \tag{12}$$

for return loss L_h at ω_h=1, choose $\sigma_o = (10^{L_h/(10n)} - 1)^{1/2}$ or

$$q = 2\,\omega_h/\sigma_o = \omega_h c/g = 2\Big/\sqrt{10^{L_h/(10n)} - 1}\,. \tag{13}$$

Choosing $\theta=\pi/(n+1)$, as in [5], the reflection response (8) of the one-port ladder network can be written as

$$\left|S_{11}(\omega^2)\right|^2 = \frac{(1 - (\omega/\sigma_o)^2)\,(\omega/\sigma_o)^{2n}}{1 - (\omega/\sigma_o)^{2(n+1)}}\,. \tag{14}$$

and, using the technique of [12], it may be shown that the coefficients of (2) and (5) for this choice of θ are

$$a_0 = \sigma_o^n \quad \text{and} \quad a_r = a_{n-r} = \frac{2\omega_h}{\sigma_o}\prod_{\mu=1}^{r}\cot\left(\mu\,\frac{\theta}{2}\right) \quad \text{for } r = 1\ldots n-1. \tag{15}$$

Rhodes [5] found explicit element value equations:

$$J_{r-1}/Y_s = 1 \tag{16}$$

$$\frac{g_r}{Y_s} = \frac{1}{E_{r-1}} - E_r \tag{17}$$

$$\frac{c_r}{Y_s} = \frac{1}{\sigma_o \cos(\theta)}\left(\frac{\sin((r-1)\theta)}{E_{r-1}} + E_r \sin(r\theta)\right) \tag{18}$$

where

$$E_0 = 1, \quad E_n = 0, \quad \text{and}$$

$$E_r E_{r-1} = \frac{\cos(\theta)+\cos(r\theta)}{\cos(\theta)+\cos((r-1)\theta)} \quad \text{for} \quad r = 1 \ldots n \tag{19}$$

and the unloaded Q of the shunt admittances is given by

$$q_r = \frac{\omega_h c_r}{g_r} = \left(\frac{2\omega_h}{\sigma_o}\right)\frac{\cos(r\theta/2)\,\cos((r-1)\theta/2)}{\sin(\theta/2)} . \tag{20}$$

The maximum unloaded Q required of the shunt admittances is given by (for r=1 in (20) or x=n+1 in (7))

$$q_1 = q_{\max} = (2\,\omega_h \,/\, \sigma_o)\cot(\theta/2). \tag{21}$$

To realize transmission poles on the unit circle in the complex s-plane, choose σ_o=1; to realize 3-dB attenuation at ω_h=1, choose σ_o to be the positive real solution of

$$2 = \sigma_o^{-2} + \sigma_o^{2n} \quad (\text{with } 1 \ge \sigma_o \ge \sqrt{2} \text{ for } n \ge 1), \tag{22}$$

or,

$$2 = \sum_{r=0}^{n} \sigma_o^{2(n-r)} \quad (\text{with } 1 \ge \sigma_o \ge \sqrt{2} \text{ for } n \ge 1); \tag{23}$$

and to realize an attenuation L_h (in dB) at ω_h=1, choose σ_o to be the positive real solution of

$$10^{L_h/10} = \sigma_o^{-2}(10^{L_h/10} - 1) + \sigma_o^{2n}, \tag{24}$$

or

$$10^{L_h/10} = \sum_{r=0}^{n} \sigma_o^{2(n-r)} . \tag{25}$$

Selectivity, k, is a useful means of comparing filter performance and, for bandstop filters, is commonly defined [13] as the ratio of two band-edge frequencies ω_s and ω_p (ω_s<ω_p) of higher and lower attenuations L_s and L_p (L_s>L_p), respectively, such that a larger k is "better":

$$k = \omega_s / \omega_p < 1. \tag{26}$$

Defining

$$\left|S_{11}(\omega_s)\right|^2 = A_s^{-2} = 10^{-L_s/10} \tag{27}$$

$$\left|S_{11}(\omega_p)\right|^2 = A_p^{-2} = 10^{-L_p/10} \tag{28}$$

allows the selectivity k_{ideal} of the ideal lossless reflection response

$$\left|S_{11}(\omega)\right|^2 = \frac{\omega^{2n}}{\omega^{2n}+1}, \tag{29}$$

the selectivity k_{equalq} of the lossy equal-q reflection response (9), and the selectivity k_{Rhodes} of Rhodes' diminishing-q reflection response (14) [5], to be expressed as follows:

$$k_{ideal} = \left(\frac{10^{L_p/10}-1}{10^{L_s/10}-1}\right)^{1/(2n)} \xrightarrow[n\to\infty]{} 1 \tag{30}$$

$$k_{equal\,q} = \left(\frac{10^{L_p/(10n)}-1}{10^{L_s/(10n)}-1}\right)^{1/2} \xrightarrow[n\to\infty]{} \sqrt{\frac{L_p}{L_s}} \tag{31}$$

$$k_{Rhodes} \approx \sqrt{\frac{1-10^{-L_p/10}}{10^{L_s/(10\,n)}}} \xrightarrow[n\to\infty]{} \sqrt{1-10^{-L_p/10}} . \tag{32}$$

from which it is evident that $k_{equal\,q} < k_{Rhodes} < k_{ideal}$. In fact, upon further inspection it is apparent that there is a continuous range of possible maximally-flat reflection-mode bandstop characteristics between that of the ideal lossless-resonator case (29) and that of the equal-Q_u-resonator case (9) corresponding to gradually decreasing selectivity and decreasing differences in finite resonator Q_u within the filter.

B. Equiripple Absorptive Notch Filters

A generic "equiripple reflection-stopband" characteristic for the prototype of Fig. 3(b) has not yet been found. However, insight is gained by examining characteristics of various lossless and lossy equiripple prototypes.

The *ideal* (lossless) "equiripple reflection-stopband" characteristic of the prototype of Fig. 3(b) is given by [e.g., 4, p. 212]:

$$S_{11}(s) = \pm\prod_{r=1}^{n} \frac{s + j\cos\theta_r}{s + (\eta\sin\theta_r + j\sqrt{1+\eta^2}\cos\theta_r)} \tag{33}$$

where

$$\theta_r = \frac{\pi}{2}\left(\frac{2r-1}{n}\right) \quad \text{for} \quad r = 1 \ldots n, \tag{34}$$

$$\eta = \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right), \tag{35}$$

$$\varepsilon = \sqrt{\frac{1}{10^{L_s/10}-1}}, \tag{36}$$

and L_s is the "reflection-stopband" minimum return loss (i.e., equiripple stopband level) in dB. The left-half-plane poles of (33) appear at

$$s_r = -(\eta\sin\theta_r + j\sqrt{1+\eta^2}\cos\theta_r) \quad \text{for} \quad r = 1 \ldots n . \tag{37}$$

These poles lie on an ellipse centered at the origin in the complex s plane, with vertical semi-major axis of extent $\sqrt{1+\eta^2}$ >1 and horizontal semi-minor axis of extent η <1.

In contrast, for the case of *lossy* admittances with equal values of g and q [8],

$$S_{11}(s) = \pm\left(\frac{s}{s+\sigma_o}\right)^{\nu} \prod_{r=1}^{(n-\nu)/2} \frac{s^2+\omega_{zr}^2}{(s+\sigma_o)^2+\omega_{zr}^2} \tag{38}$$

where $\nu = (1-(-1)^n)/2$, $\sigma_o = 2\omega_h/q$, and values of q and w_r=ω_{zr}/ω_h are given in Table II for highpass prototypes of orders n=2 to 26 with equiripple return loss L_s (=L_h) ranging from 20-dB to 85-dB in 5-dB increments. Note that for all odd n, there is a reflection zero at ω=0 and, for all n, there are reflection zeros at $\omega = \pm\omega_{zr} = \pm w_r\,\omega_h$. The left-half-plane poles of (38) appear at

$$-(\sigma_o \pm j\,\omega_{zr}) \quad \text{for} \quad r = 1\ldots(n-\nu)/2\,, \tag{39}$$

with an additional pole at $-\sigma_o$ for odd n. Instead of lying on an ellipse, as in the lossless equiripple case, or collapsing to the point $s=-\sigma_o$, as in the maximally flat equal-q case, the left-half-plane reflection poles in the equiripple equal-q case lie on the vertical line through the point $s=-\sigma_o=2\omega_h/q$.

Like the lossy maximally flat reflection prototype characteristics, a continuum of possible equiripple characteristics – of lossy reflection prototypes whose shunt admittances exhibit progressively diminishing q – exists between the characteristics of the **ideal lossless** equiripple prototypes and those of the **equal-q lossy** equiripple prototypes. In [5], Rhodes describes an approximation to one such set of intermediate characteristics, for which

$$S_{11}(s) = \pm\prod_{r=1}^{n} \frac{s + j\sigma_o\,\alpha^{-1}\cos\theta_r}{s + (\sigma_o\sqrt{1-\alpha^{-2}}\,\sin\theta_r + j\,\sigma_o\cos\theta_r)} \tag{40}$$

where

$$\theta_r = \frac{\pi}{2}\left(\frac{2r}{n+1}\right) \quad \text{for} \quad r = 1\ldots n\,, \tag{41}$$

$$\alpha = 10^{-L_s/20}\,T_{n+1}(\alpha)\,, \tag{42}$$

$T_{n+1}(\alpha)$ is the Chebyshev polynomial of degree $n+1$ and argument α, σ_o is an ***empirical*** constant that scales the bandwidth, and L_s ($=L_h$) is the "reflection-stopband" *nominal* minimum return loss (i.e., *quasi-equiripple* stopband level) in dB. The left-half-plane poles of (40) are

$$s_r = -(\sigma_o\sqrt{1-\alpha^{-2}}\,\sin\theta_r + j\,\sigma_o\cos\theta_r) \quad \text{for} \quad r = 1\ldots n\,. \tag{43}$$

Plotting the reflection poles of the ideal lossless, equal-q lossy, and Rhodes' lossy prototypes, as shown in Fig. 5, suggests that the reflection poles of a generic lossy equiripple prototype fall within the bounding boxes defined by the corresponding lossless and lossy-equal-q poles – as illustrated by the shaded bounding boxes in Fig. 5. It is also evident from Fig. 5 that the one-port prototype's reflection poles move from an ellipse in the lossless case to the dashed vertical line through the point $s=-\sigma_o=2\omega_h/q$ in the equal-q case (as the frequency of the reflection poles gradually becomes equal to the frequency of the reflection zeros), and lie on an ellipse of decreasing horizontal minor axis and increasing vertical major axis for diminishing values of q_{max}.

The lossy equal-q prototype may be synthesized by referring to (10) for J_0 and the design data in Table III, which provides normalized admittance inverter values $J_r=J_r/g$ for $r>0$ for filters of order $n=2$ to 21 and "reflection- stopband" attenuations $L_s=L_h=20$-dB to 85-dB in 5-dB increments, as well as the q and "selectivity" data in Table IV, which provides ratios of the normalized highpass prototype passband-edge frequencies to

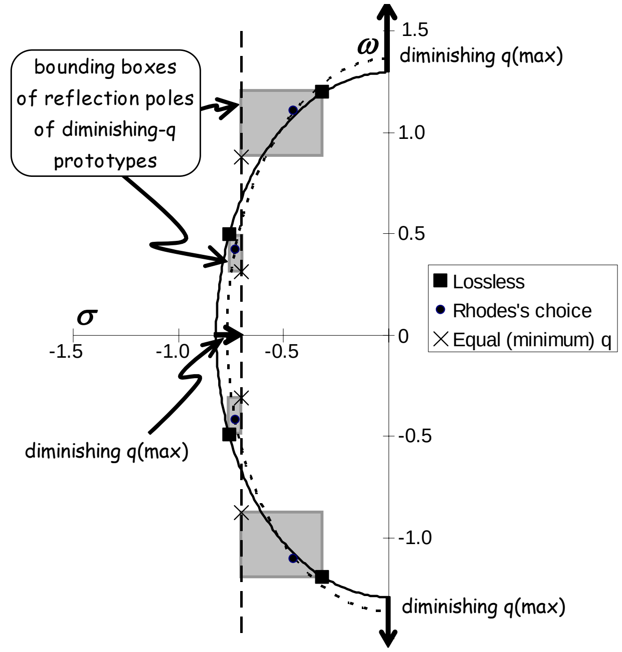

Fig. 5. Prototype LHP equiripple reflection poles for $n=4$, plotted in the complex $s=\sigma+j\omega$ plane, for three different sets of shunt admittance unloaded Qs, showing the trend in the semi-major and semi-minor axes of the ellipses through the poles as the maximum q of the shunt admittances diminishes.

stopband-edge frequencies for passband-edge insertion loss levels of 3.01-dB, 2-dB, 1-dB, and 0.5-dB and for equiripple "reflection-stopband" attenuation levels of $L_s=L_h=20$-dB to 85-dB in 5-dB increments. The equal-q shunt admittances are given by $q=(\omega_h\ c_r\ /g_r)$, and the corresponding equal-q reflection response is given by [8]

$$\left|S_{11}(\omega)\right|^2 = \left(\frac{\omega^2}{\omega^2+\sigma_o^2}\right)^{\nu} \prod_{r=1}^{(n-\nu)/2} \frac{(\omega^2-\omega_{zr}^2)^2}{((\omega-\omega_{zr})^2+\sigma_o^2)\,((\omega+\omega_{zr})^2+\sigma_o^2)} \tag{44}$$

where ν, σ_o, q, and ω_{zr} are the same as in (38).

The approximate equiripple lossy filters of Rhodes may be synthesized using (41), (42), and the element value equations [5]

$$\frac{J_0}{Y_s} = \sqrt{\frac{\sqrt{\alpha^2-1}}{\alpha}}\,, \tag{45}$$

$$\frac{J_r}{Y_s} = \frac{\sqrt{\alpha^2-\cos^2(\theta_r)}}{\alpha} \quad \textit{for} \quad r = 1\ldots n-1\,, \tag{46}$$

with

$$\frac{g_r}{Y_s} = \frac{\sqrt{\alpha^2-1}}{\alpha}\left(\frac{1}{E_{r-1}} - E_r\right), \tag{47}$$

c_r as in (18) (but, with σ_o empirically derived to give the target filter bandwidth), E_r as in (19), and $q_r=(\omega_h\ c_r\ /g_r)$. The corresponding reflection response is given by [5]

$$\left|S_{11}(\omega)\right|^2 = \left(\frac{1-T_{n+1}^2(\alpha\,\omega/\sigma_o)}{1-(\alpha\,\omega/\sigma_o)^2}\right)\left(\frac{\alpha^2-(\alpha\,\omega/\sigma_o)^2}{T_{n+1}^2(\alpha)-T_{n+1}^2(\alpha\,\omega/\sigma_o)}\right). \tag{48}$$

III. Design Examples

The highpass reflection prototype of a fourth-order absorptive (reflection-mode) bandstop filter is designed in five different ways: (a) as an equal-Q_u maximally-flat filter, (b) as a moderately-ranging-Q_u (Rhodes') maximally-flat filter, (c) as a "generic" widely-ranging-Q_u maximally-flat filter, (d) as an equal-Q_u equiripple filter, and (e) as a moderately-ranging-Q_u (Rhodes') equiripple filter. Specifying a "reflection-stopband" level of L_h=45 dB at ω_h=±1 and a source impedance of $R_s=1/Y_s$=50 ohms, the individual designs proceed as described below.

For case (a), the equal-q maximally-flat highpass reflection prototype design begins by calculating q from (13) as q=0.569451, arbitrarily choosing a value for c of c=1, and calculating g as $g=\omega_h\, c\,/q=1/q$=1.75608. Then the admittance inverter values are calculated to be J_0=0.374815 from (10) and J_1=3.92671, J_2=1.57068, and J_3=0.785341 from (11). The reflection response is given by (9) and σ_o as calculated from $\sigma_o=2\omega_h/q$=3.51215.

For case (b), the design of the moderately-ranging-q maximally-flat highpass reflection prototype of Rhodes [5] begins by calculating the admittance inverter values from (16) as $J_0= J_1= J_2= J_3=Y_s$=0.02, determining x=5 and θ=0.628319 from the third row of Table I, and calculating E_r from (17) as E_0=1, E_1=0.894427, E_2=0.772542, E_3=0.578885, and E_4=0. The shunt conductance values are then found from (17) to be g_1=0.00211146, g_2= 0.00690983, g_3=0.00143108, and g_4=0.0345492, while the shunt capacitance values are found by first solving (25) for σ_o to find σ_o=3.6156 and then using (18) to calculate c_1=0.00359465, c_2=0.00951698, c_3=0.0121817, and c_4=0.0112333. Shunt admittance q's are found from (20) to be q_1=1.70245, q_2=1.37731, q_3=0.851224, and q_4=0.325139 and the reflection response is given by (14).

For case (c), the design of the "generic" widely-ranging-q maximally-flat highpass reflection prototype begins by assuming some maximum realizable shunt admittance q – in this case, q_{max}=10 – and an initial value of $x=n=4$. Three iterations of first solving the generic maximally-flat reflection characteristic (2) for σ_o (such that 20 $\log_{10}|S_{11}(j\omega_h)|= -L_h$) and then solving (7) for a new value of x results in values of x and σ_o for q_{max}=10 of x=4.13412 and σ_o=3.64637. The coefficients of the one-port input admittance (5) are then found from (6) to be a_0=176.784, a_1=130.551, a_2=47.2229, and a_3=9.81884. Successively extracting normalized unity admittance inverters, shunt capacitors, and shunt conductances [e.g., 11] from (5) and then denormalizing by scaling by Y_s=1/50 results in admittance inverters $J_0=J_1=J_2=J_3$=0.02, shunt capacitances c_1=0.0040738, c_2=0.00998408, c_3=0.0104151, and c_4=0.00514398, shunt conductances g_1=0.000407387, g_2=0.00137734, g_3=0.00319442, and g_4=0.0224522, and shunt admittance q's q_1=9.99984, q_2=7.24881, q_3=3.2604, and q_4=0.229108. The reflection response is given by (2).

For case (d), the equal-q equiripple highpass reflection prototype design begins by choosing an arbitrary value for shunt conductance g – in this case, g=2. The first admittance inverter J_0 is found, using (10), to be J_0=0.4. The q of the shunt admittances and the remaining normalized admittance inverter values are found from Table III (for n=4 and $L_s=L_h$=45): q=1.01972, J_1=2.34454, J_2=1.05528, and J_3=0.619077. After scaling by g ($J_r=J_r\, g$), the corresponding admittance inverter values are J_1=4.68908, J_2=2.11056, and J_3=1.23815. The shunt capacitance values can be calculated using $c=qg/\omega_h$. The reflection response can be calculated using (44), $\sigma_o = 2\omega_h/q$, and $\omega_{zr}=\omega_h w_r$, with w_1=0.354843 and w_2=0.910914 from Table II.

Finally, for case (e), Rhodes moderately-ranging-q equiripple highpass reflection prototype design [5] can begin by calculating a=1.99261 from (42), J_0=0.0186006 from (45), and J_1=0.0182774 and $J_2=J_3$=0.019758 from (46). Values of E_r as given for case (b), and calculated using (19), are employed in (47) to calculate values of shunt conductance g_1=0.00182631, g_2=0.00597667, g_3=0.0123782, and g_4=0.0298834. Solving the equiripple reflection characteristic (48) for σ_o (such that 10 $\log_{10}|S_{11}(\omega_h)|^2= -L_h$) gives σ_o=2.28686, which is used in (18) to calculate c_1=0.00568324, c_2=0.0150466, c_3=0.0192597, c_4=0.0177601. The values of shunt admittance q_r ($=\omega_h\, c_r/g_r$) are q_1=3.11187, q_2=2.51756, q_3=1.55594, and q_4=0.594315. And, the reflection response is given by (48).

Resulting values of q_r and J_{r-1} for each case are collected in Table V, and the reflection responses of the corresponding prototypes are compared in Figs. 6-8.

case:	(a)	(b)	(c)	(d)	(e)
q_1	0.569451	1.70245	9.99984	1.01972	3.11187
q_2	0.569451	1.37731	7.24881	1.01972	2.51756
q_3	0.569451	0.851224	3.2604	1.01972	1.55594
q_4	0.569451	0.325139	0.229108	1.01972	0.594315
J_0	0.374815	0.02	0.02	0.4	0.018601
J_1	3.92671	0.02	0.02	4.68908	0.018277
J_2	1.57068	0.02	0.02	2.11056	0.01976
J_3	0.785341	0.02	0.02	1.23815	0.01976

Table V. Collected values of q_r and J_{r-1} for the prototype circuit of Fig. 3(b) for design examples (a) - (e).

IV. Conclusion

The synthesis of lossy maximally-flat and equiripple highpass reflection-mode prototypes for application to the design of lossy reflection-mode bandstop filters has been

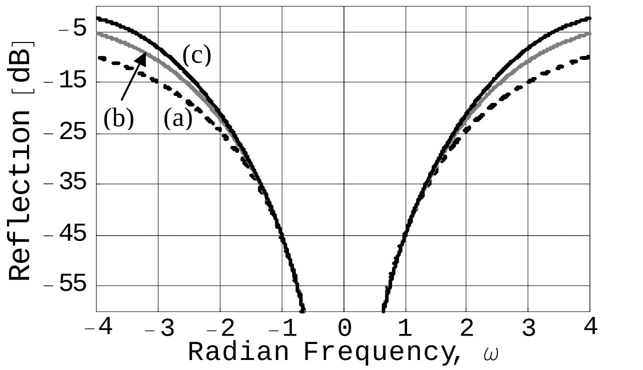

Fig. 6. A comparison of the reflection responses of maximally-flat examples (a) (dashed), (b) (solid gray), and (c) (solid black).

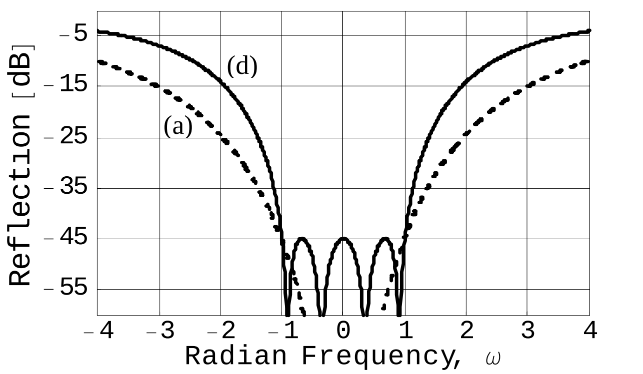

Fig. 7. A comparison of the prototype reflection responses of equal-q maximally-flat design example (a) (dashed curve) and equal-q equiripple design example (d) (solid curve).

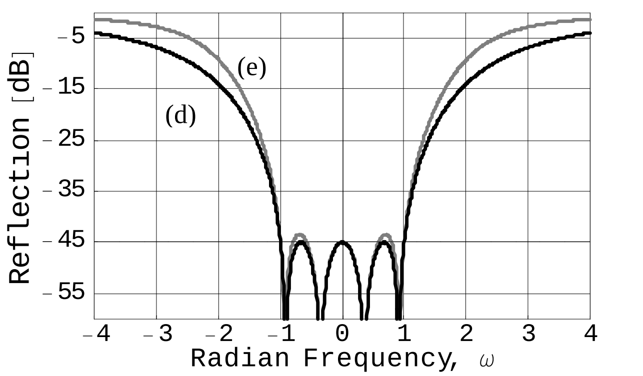

Fig. 8. A comparison of the prototype reflection responses of equiripple design example (d) (solid black curve) and quasi-equiripple design example (e) (solid gray curve).

described. Extensive design tables for lossy equal-q equiripple highpass reflection prototypes are included. Specific examples of the use of the synthesis methods have been provided for equal-q, Rhodes' progressively-diminishing-q, and generalized maximally-flat highpass reflection prototypes, as well as equal-q and Rhodes' progressively-diminishing-q equiripple highpass reflection prototypes. Tables of representative admittance inverter and shunt admittance q values, and plots of representative reflection characteristics of each type of filter, allow for comparisons of the relative characteristics of the different types of designs and an assessment of their relative advantages and disadvantages for various application requirements. These types of filters are potentially suited for interference suppression in receiver front-ends.

APPENDIX A. *Derivation of (6)*

The derivation of (6) follows Linnebach's technique, as described by Bosse in [12]. The reflection characteristic $S_{11}(s)$ in (2) can be written as

$$S_{11}(s) = N(s)/D(s) \tag{A.1}$$

where

$$N(s) = \pm s^n \tag{A.2}$$

$$D(s) = \prod_{r=1}^{n} (s - j\,\sigma_o\, e^{j\theta_r})\,. \tag{A.3}$$

Using (4), (A.3) can be rewritten as

$$D(s) = \prod_{r=1}^{n} \left(s - j\,\sigma_o\, e^{j\frac{\pi}{2}\left(\frac{2r+x-(n+1)}{x}\right)} \right) \tag{A.4}$$

$$= \prod_{r=1}^{n} \left(s + \sigma_o\, e^{j\frac{\pi}{2}\left(\frac{2r-(n+1)}{x}\right)} \right).$$

Extracting the factor for r=1 from the product and increasing the range of the product by one gives

$$D(s) = \frac{s+\sigma_o\, e^{j\frac{\pi}{2}\left(\frac{1-n}{x}\right)}}{s+\sigma_o\, e^{\frac{\pi}{2}\left(\frac{1+n}{x}\right)}} \prod_{r=2}^{n+1} \left(s + \sigma_o\, e^{j\frac{\pi}{2}\left(\frac{2r-(n+1)}{x}\right)} \right). \tag{A.5}$$

And, changing variables, such that r=r'+1, results in

$$D(s) = \frac{s+\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n}{x}\right)}}{s+\sigma_o e^{\frac{\pi}{2}\left(\frac{1+n}{x}\right)}} \prod_{r'=1}^{n} \left(s + \sigma_o e^{j\frac{\pi}{2}\left(\frac{2(r'+1)-(n+1)}{x}\right)} \right) \tag{A.6}$$

$$= \frac{s-j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}}{s+j\sigma_o e^{\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}} \prod_{r'=1}^{n} \left(s + \sigma_o e^{j\frac{\pi}{x}} e^{j\frac{\pi}{2}\left(\frac{2r'-(n+1)}{x}\right)} \right)$$

$$= \frac{s-j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}}{s+j\sigma_o e^{\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}}\, e^{j\frac{\pi n}{x}} \prod_{r'=1}^{n} \left(s\, e^{-j\frac{\pi}{x}} + \sigma_o e^{j\frac{\pi}{2}\left(\frac{2r'-(n+1)}{x}\right)} \right).$$

Now, letting r'=r in (A.6) and comparing this to (A.4) suggests that (A.6) can be written as

$$D(s) = \frac{s-j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}}{s+j\sigma_o e^{\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}}\, e^{j\frac{\pi n}{x}}\, D\left(s\, e^{-j\frac{\pi}{x}} \right). \tag{A.7}$$

Since, by definition,

$$0 = D(s) - D(s) \tag{A.8}$$

$$= D(s) - \frac{s - j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}}{s + j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}} e^{j\frac{\pi n}{x}} D(s\, e^{-j\frac{\pi}{x}})$$

$$= \left(s + j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}\right) D(s) - \left(s - j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}\right) e^{j\frac{\pi n}{x}} D(s\, e^{-j\frac{\pi}{x}})$$

and, noting from (2) that

$$D(s) = \sum_{\lambda=0}^{n} a_\lambda\, s^\lambda \tag{A.9}$$

$$D(s\, e^{-j\frac{\pi}{x}}) = \sum_{\lambda=0}^{n} a_\lambda\, s^\lambda\, e^{-j\frac{\pi}{x}\lambda} \tag{A.10}$$

it is apparent that

$$0 = \left(s + j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)}\right) \sum_{\lambda=0}^{n} a_\lambda\, s^\lambda - \left(s - j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1-n+x}{x}\right)}\right) e^{j\frac{\pi n}{x}} \sum_{\lambda=0}^{n} a_\lambda\, s^\lambda\, e^{-j\frac{\pi}{x}\lambda}. \tag{A.11}$$

A recursion formula can be developed by comparing the coefficients of equivalently high powers of s in (A.11) with one another. Equating the coefficients of the $s^{\lambda+1}$ terms in (A.11) results in

$$0 = \left(a_\lambda + j\sigma_o e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)} a_{\lambda+1}\right) - \left(e^{j\frac{\pi}{x}(n-\lambda)} a_\lambda - j\sigma_o e^{-j\frac{\pi}{2}\left(\frac{1-n-x+2\lambda}{x}\right)} a_{\lambda+1}\right) \tag{A.12}$$

$$= a_\lambda \left(1 - e^{j\frac{\pi}{x}(n-\lambda)}\right) + j\, a_{\lambda+1}\, \sigma_o \left(e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)} + e^{-j\frac{\pi}{2}\left(\frac{1-n-x+2\lambda}{x}\right)}\right)$$

which gives the recurrence relation

$$\frac{a_{\lambda+1}}{a_\lambda} = \left(\frac{1}{j\sigma_o}\right) \frac{e^{j\frac{\pi}{x}(n-\lambda)} - 1}{e^{j\frac{\pi}{2}\left(\frac{1+n-x}{x}\right)} + e^{-j\frac{\pi}{2}\left(\frac{1-n-x+2\lambda}{x}\right)}}. \tag{A.13}$$

(A.13) can then be put into the form

$$\frac{a_{\lambda+1}}{a_\lambda} = \left(\frac{1}{\sigma_o}\right) \frac{\left(\dfrac{e^{j\frac{\pi}{2}\left(\frac{n-\lambda}{x}\right)} - e^{-j\frac{\pi}{2}\left(\frac{n-\lambda}{x}\right)}}{2j}\right)}{\left(\dfrac{e^{j\frac{\pi}{2}\left(\frac{1+\lambda-x}{x}\right)} + e^{-j\frac{\pi}{2}\left(\frac{1+\lambda-x}{x}\right)}}{2}\right)} \tag{A.14}$$

Applying well-known trigonometric identities [15] to (A.14) leads to a variety of equivalent recurrence relations:

$$\frac{a_{\lambda+1}}{a_\lambda} = \left(\frac{1}{\sigma_o}\right) \frac{\sin\left(\frac{\pi}{2}\left(\frac{n-\lambda}{x}\right)\right)}{\cos\left(\frac{\pi}{2}\left(\frac{x-(\lambda+1)}{x}\right)\right)} \tag{A.15}$$

$$\frac{a_{\lambda+1}}{a_\lambda} = \left(\frac{1}{\sigma_o}\right) \frac{\sin\left(\frac{\pi}{2}\left(\frac{n-\lambda}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{\lambda+1}{x}\right)\right)} \tag{A.16}$$

$$\frac{a_{\lambda+1}}{a_\lambda} = \left(\frac{1}{\sigma_o}\right) \frac{\cos\left(\frac{\pi}{2}\left(\frac{\lambda+x-n}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{\lambda+1}{x}\right)\right)} \tag{A.17}$$

By expanding out (5), with S_{11} as in (2), for various values of n and x, it can be demonstrated that

$$a_0 = \sigma_o^n. \tag{A.18}$$

Using (A.16) and (A.18), the next two coefficients are

$$a_1 = \sigma_o^{(n-1)} \frac{\sin\left(\frac{\pi}{2}\left(\frac{n}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{1}{x}\right)\right)} \tag{A.19}$$

$$a_2 = \sigma_o^{(n-2)} \frac{\sin\left(\frac{\pi}{2}\left(\frac{n}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{1}{x}\right)\right)} \left(\frac{\sin\left(\frac{\pi}{2}\left(\frac{n-1}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{2}{x}\right)\right)}\right) \tag{A.20}$$

from which it is apparent that

$$a_r = \sigma_o^{(n-r)} \prod_{\mu=1}^{r} \frac{\sin\left(\frac{\pi}{2}\left(\frac{n-(\mu-1)}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{\mu}{x}\right)\right)}. \tag{A.21}$$

Again, applying well-known trigonometric identities ($\cos\theta = \sin(\pi/2 \pm \theta)$ and $\cos(\pi/2 \pm \theta) = \pm\sin\theta$) [15] to (A.21) leads to

$$a_r = \sigma_o^{(n-r)} \prod_{\mu=1}^{r} \frac{\cos\left(\frac{\pi}{2}\left(\frac{x+\mu-(n+1)}{x}\right)\right)}{\sin\left(\frac{\pi}{2}\left(\frac{\mu}{x}\right)\right)}. \qquad \text{(A.22)}$$

Finally, since the denominator of (A.21) with $\mu \to n-\mu$ is equal to the numerator of (A.21) with $\mu \to \mu+1$, i.e.,

$$\sin\left(\frac{\pi}{2x}(n-\mu)\right) = \sin\left(\frac{\pi}{2x}(n-((\mu+1)-1)\right), \qquad \text{(A.23)}$$

then

$$a_r = \sigma_o^{(n-2r)} a_{n-r}. \qquad \text{(A.24)}$$

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n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
2	q	0.984726	0.708574	0.518897	0.383886	0.285667	0.213294	0.159557	0.119486	0.0895321	0.0671103	0.0503132	0.0377244	0.0282876	0.0212116
	$w1$	0.677000	0.689016	0.696515	0.701010	0.703621	0.705134	0.705994	0.706480	0.706754	0.706908	0.706995	0.707044	0.707083	0.707094
3	q	1.89529	1.44730	1.13210	0.900258	0.724114	0.587102	0.478657	0.391752	0.321481	0.264295	0.217560	0.179243	0.147761	0.121857
	$w1$	0.820110	0.832782	0.842249	0.849209	0.854238	0.857828	0.860354	0.862119	0.863343	0.864182	0.864766	0.865166	0.865439	0.865626
4	q	2.84833	2.23007	1.79171	1.46629	1.21650	1.01972	0.861435	0.732049	0.624943	0.535355	0.459830	0.395744	0.341115	0.294361
	$w1$	0.311169	0.321199	0.330849	0.339828	0.347868	0.354843	0.360718	0.365548	0.369433	0.372517	0.374914	0.376782	0.378209	0.379302
	$w2$	0.876702	0.887298	0.895557	0.901979	0.906994	0.910914	0.913955	0.916304	0.918119	0.919508	0.920575	0.921379	0.921996	0.922460
5	q	3.82375	3.03559	2.47456	2.05627	1.73353	1.47792	1.27116	1.10101	0.959002	0.839083	0.736851	0.648979	0.572936	0.506761
	$w1$	0.478158	0.490507	0.502468	0.513834	0.524398	0.534042	0.542660	0.550225	0.556755	0.562334	0.567024	0.570926	0.574153	0.576807
	$w2$	0.906619	0.915453	0.922428	0.927958	0.932376	0.935938	0.938825	0.941171	0.943080	0.944627	0.945887	0.946909	0.947733	0.948395
6	q	4.81317	3.85527	3.17162	2.66061	2.26532	1.95121	1.69626	1.48575	1.30940	1.15986	1.03176	0.921034	0.824602	0.740075
	$w1$	0.195726	0.200413	0.205072	0.209719	0.214308	0.218822	0.223195	0.227362	0.231273	0.234886	0.238172	0.241110	0.243733	0.246026
	$w2$	0.581229	0.593684	0.605685	0.617144	0.627873	0.637827	0.646922	0.655136	0.662477	0.668975	0.674679	0.679627	0.683930	0.687615
	$w3$	0.925026	0.932539	0.938494	0.943251	0.947101	0.950239	0.952822	0.954969	0.956760	0.958260	0.959520	0.960579	0.961463	0.962204
7	q	5.81230	4.68447	3.87850	3.27483	2.80692	2.43447	2.13154	1.88084	1.67035	1.49136	1.33764	1.20439	1.08803	0.985618
	$w1$	0.328840	0.335666	0.342404	0.349095	0.355706	0.362220	0.368595	0.374774	0.380692	0.386320	0.391590	0.396495	0.400954	0.405035
	$w2$	0.650853	0.662771	0.674193	0.685079	0.695281	0.704793	0.713557	0.721569	0.728834	0.735392	0.741269	0.746522	0.751156	0.755261
	$w3$	0.937460	0.943966	0.949143	0.953288	0.956656	0.959424	0.961722	0.963649	0.965280	0.966658	0.967839	0.968845	0.969722	0.970465
8	q	6.81816	5.52077	4.59231	3.89604	3.35570	2.92495	2.57406	2.28325	2.03867	1.83042	1.65123	1.49559	1.35938	1.23931
	$w1$	0.142157	0.144868	0.147548	0.150207	0.152854	0.155488	0.158114	0.160721	0.163298	0.165827	0.168289	0.170667	0.172939	0.175090
	$w2$	0.424856	0.432611	0.440246	0.447773	0.455204	0.462503	0.469665	0.476636	0.483374	0.489833	0.495977	0.501780	0.507205	0.512246
	$w3$	0.700947	0.712113	0.722802	0.732939	0.742471	0.751353	0.759566	0.767109	0.774003	0.780275	0.785960	0.791098	0.795723	0.799877
	$w4$	0.946395	0.952134	0.956698	0.960361	0.963344	0.965804	0.967855	0.969584	0.971054	0.972314	0.973401	0.974339	0.975156	0.975869
9	q	7.82927	6.36200	5.31132	4.52265	3.90997	3.42089	3.02213	2.69124	2.41257	2.17515	1.97051	1.79255	1.63657	1.49891
	$w1$	0.249659	0.253949	0.258181	0.262358	0.266502	0.270619	0.274715	0.278781	0.282823	0.286796	0.290710	0.294534	0.298235	0.301801
	$w2$	0.497217	0.505316	0.513265	0.521064	0.528735	0.536264	0.543637	0.550812	0.557783	0.564473	0.570895	0.577007	0.582777	0.588207
	$w3$	0.738628	0.749050	0.759006	0.768422	0.777270	0.785509	0.793139	0.800160	0.806606	0.812491	0.817862	0.822747	0.827180	0.831204
	$w4$	0.953125	0.958247	0.962325	0.965604	0.968278	0.970484	0.972329	0.973890	0.975218	0.976370	0.977362	0.978225	0.978983	0.979650
10	q	8.84453	7.20765	6.03450	5.15344	4.46830	3.92100	3.47449	3.10354	2.79091	2.52422	2.29419	2.09394	1.91827	1.76304
	$w1$	0.111414	0.113197	0.114953	0.116687	0.118404	0.120112	0.121809	0.123502	0.125191	0.126875	0.128550	0.130211	0.131851	0.133463
	$w2$	0.333628	0.338840	0.343960	0.349008	0.353997	0.358945	0.363843	0.368708	0.373527	0.378296	0.382991	0.387597	0.392096	0.396465
	$w3$	0.553634	0.561776	0.569734	0.577529	0.585167	0.592654	0.599959	0.607077	0.613975	0.620640	0.627032	0.633138	0.638948	0.644450
	$w4$	0.767981	0.777712	0.786974	0.795728	0.803937	0.811586	0.818665	0.825189	0.831181	0.836679	0.841704	0.846291	0.850482	0.854305
	$w5$	0.958370	0.962996	0.966678	0.969642	0.972058	0.974055	0.975732	0.977149	0.978362	0.979410	0.980320	0.981114	0.981815	0.982433
11	q	9.86305	8.05652	6.76115	5.78756	5.03004	4.42461	3.93022	3.51929	3.17269	2.87675	2.62137	2.39891	2.20355	2.03077
	$w1$	0.200904	0.203869	0.206782	0.209651	0.212487	0.215298	0.218091	0.220871	0.223640	0.226401	0.229145	0.231875	0.234582	0.237256
	$w2$	0.400944	0.406682	0.412307	0.417834	0.423283	0.428669	0.434000	0.439279	0.444503	0.449670	0.454756	0.459757	0.464654	0.469429
	$w3$	0.598810	0.606837	0.614662	0.622302	0.629770	0.637072	0.644195	0.651123	0.657837	0.664322	0.670549	0.676514	0.682202	0.687607
	$w4$	0.791474	0.800573	0.809216	0.817367	0.825003	0.832112	0.838696	0.844765	0.850345	0.855467	0.860161	0.864460	0.868394	0.871995
	$w5$	0.962570	0.966785	0.970142	0.972843	0.975047	0.976872	0.978402	0.979699	0.980810	0.981770	0.982608	0.983343	0.983990	0.984564

Table II. Prototype values of q and $w_r = \omega_{zr}/\omega_h$ for absorptive bandstop filters of order n with equal resonator Q_u and equiripple stopband attenuation L_s=L_h.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
12	q	10.8844	8.90817	7.49048	6.42448	5.59459	4.93107	4.38871	3.93779	3.55729	3.23220	2.95147	2.70677	2.49171	2.30147
	$w1$	0.0915195	0.0927903	0.0940380	0.0952661	0.0964786	0.0976800	0.0988713	0.100057	0.101239	0.102418	0.103594	0.104768	0.105939	0.107104
	$w2$	0.274275	0.278025	0.281703	0.285320	0.288886	0.292416	0.295912	0.299386	0.302843	0.306283	0.309705	0.313106	0.316483	0.319824
	$w3$	0.456071	0.462091	0.467978	0.473752	0.479430	0.485034	0.490561	0.496029	0.501434	0.506769	0.512027	0.517195	0.522260	0.527206
	$w4$	0.635780	0.643610	0.651223	0.658640	0.665875	0.672938	0.679809	0.686489	0.692961	0.699206	0.705216	0.710972	0.716468	0.721702
	$w5$	0.810693	0.819223	0.827309	0.834925	0.842050	0.848683	0.854816	0.860475	0.865683	0.870466	0.874856	0.878880	0.882569	0.885957
	$w6$	0.966007	0.969879	0.972961	0.975442	0.977467	0.979145	0.980551	0.981746	0.982770	0.983657	0.984431	0.985111	0.985710	0.986245
13	q	11.9080	9.76204	8.22212	7.06371	6.16150	5.43966	4.84964	4.35874	3.94430	3.59001	3.28394	3.01704	2.78237	2.57460
	$w1$	0.167948	0.170128	0.172267	0.174370	0.176442	0.178490	0.180521	0.182538	0.184545	0.186544	0.188538	0.190527	0.192510	0.194486
	$w2$	0.335475	0.339745	0.343927	0.348032	0.352073	0.356063	0.360012	0.363928	0.367816	0.371680	0.375521	0.379336	0.383122	0.386872
	$w3$	0.502018	0.508164	0.514166	0.520041	0.525807	0.531481	0.537078	0.542599	0.548049	0.553422	0.558716	0.563916	0.569014	0.573997
	$w4$	0.666579	0.674168	0.681537	0.688700	0.695675	0.702466	0.709074	0.715486	0.721694	0.727682	0.733443	0.738966	0.744243	0.749273
	$w5$	0.826701	0.834719	0.842309	0.849447	0.856119	0.862321	0.868062	0.873356	0.878228	0.882704	0.886816	0.890591	0.894057	0.897241
	$w6$	0.968872	0.972450	0.975299	0.977593	0.979465	0.981017	0.982319	0.983425	0.984374	0.985196	0.985915	0.986547	0.987106	0.987603
14	q	12.9335	10.6179	8.95572	7.70485	6.73036	5.95035	5.31252	4.78167	4.33329	3.94988	3.61845	3.32931	3.07506	2.84979
	$w1$	0.0776109	0.0785667	0.0795037	0.0804239	0.0813307	0.0822263	0.0831133	0.0839937	0.0848689	0.0857408	0.0866098	0.0874770	0.0883429	0.0892072
	$w2$	0.232684	0.235520	0.238297	0.241023	0.243708	0.246357	0.248979	0.251580	0.254164	0.256736	0.259296	0.261848	0.264391	0.266924
	$w3$	0.387271	0.391882	0.396391	0.400809	0.405154	0.409435	0.413665	0.417854	0.422006	0.426128	0.430219	0.434279	0.438307	0.442296
	$w4$	0.540884	0.547060	0.553081	0.558964	0.564730	0.570394	0.575970	0.581465	0.586879	0.592214	0.597462	0.602616	0.607670	0.612609
	$w5$	0.692621	0.699957	0.707066	0.713964	0.720672	0.727193	0.733528	0.739672	0.745612	0.751343	0.756852	0.762134	0.767187	0.772005
	$w6$	0.840235	0.847795	0.854940	0.861651	0.867920	0.873743	0.879131	0.884099	0.888671	0.892874	0.896735	0.900281	0.903543	0.906541
	$w7$	0.971295	0.974622	0.977270	0.979401	0.981143	0.982585	0.983796	0.984826	0.985710	0.986476	0.987146	0.987735	0.988258	0.988723
15	q	13.9607	11.4755	9.69102	8.34776	7.30086	6.46276	5.77712	5.20631	4.72403	4.31146	3.95471	3.64338	3.36945	3.12673
	$w1$	0.144211	0.145889	0.147533	0.149145	0.150731	0.152297	0.153846	0.155382	0.156907	0.158424	0.159935	0.161441	0.162943	0.164443
	$w2$	0.288193	0.291500	0.294736	0.297908	0.301026	0.304102	0.307141	0.310151	0.313139	0.316107	0.319059	0.321999	0.324925	0.327839
	$w3$	0.431661	0.436487	0.441201	0.445814	0.450341	0.454799	0.459196	0.463544	0.467849	0.472116	0.476346	0.480541	0.484697	0.488814
	$w4$	0.574179	0.580322	0.586302	0.592136	0.597844	0.603446	0.608950	0.614368	0.619700	0.624947	0.630105	0.635169	0.640128	0.644979
	$w5$	0.714926	0.722004	0.728853	0.735490	0.741931	0.748187	0.754257	0.760138	0.765821	0.771300	0.776565	0.781614	0.786442	0.791050
	$w6$	0.851825	0.858973	0.865720	0.872050	0.877957	0.883442	0.888514	0.893191	0.897495	0.901452	0.905088	0.908430	0.911502	0.914331
	$w7$	0.973371	0.976479	0.978952	0.980943	0.982570	0.983918	0.985050	0.986012	0.986839	0.987555	0.988182	0.988735	0.989224	0.989661
16	q	14.9895	12.3326	10.4264	8.99088	7.87289	6.97576	6.24323	5.63247	5.11628	4.67456	4.29250	3.95902	3.66542	3.40527
	$w1$	0.0673484	0.0680907	0.0688238	0.0695415	0.0702524	0.0709448	0.0716377	0.0723200	0.0729971	0.0736702	0.0743402	0.0750081	0.0756735	0.0763385
	$w2$	0.201960	0.204168	0.206349	0.208483	0.210596	0.212651	0.214709	0.216734	0.218742	0.220738	0.222723	0.224702	0.226671	0.228637
	$w3$	0.336298	0.339915	0.343483	0.346970	0.350421	0.353773	0.357127	0.360423	0.363690	0.366932	0.370153	0.373357	0.376541	0.379714
	$w4$	0.470115	0.475033	0.479871	0.484593	0.489255	0.493779	0.498295	0.502726	0.507107	0.511444	0.515739	0.519997	0.524206	0.528379
	$w5$	0.603015	0.609044	0.614951	0.620697	0.626348	0.631821	0.637257	0.642568	0.647790	0.652923	0.657964	0.662913	0.667752	0.672491
	$w6$	0.734240	0.741030	0.747630	0.754010	0.760226	0.766200	0.772045	0.777676	0.783113	0.788352	0.793385	0.798213	0.802824	0.807233
	$w7$	0.861861	0.868615	0.875006	0.880993	0.886595	0.891760	0.896567	0.900983	0.905046	0.908782	0.912215	0.915374	0.918276	0.920953
	$w8$	0.975170	0.978080	0.980401	0.982269	0.983798	0.985060	0.986125	0.987029	0.987805	0.988478	0.989067	0.989587	0.990047	0.990458

Table II. Prototype values of q and $w_r = \omega_{zr}/\omega_h$ for absorptive bandstop filters of order n with equal resonator Q_u and equiripple stopband attenuation L_s=L_h.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
17	q	16.0194	13.1948	11.1658	9.63774	8.44623	7.49181	6.71063	6.05992	5.50981	5.03896	4.63159	4.27587	3.96271	3.68506
	$w1$	0.126315	0.127649	0.128955	0.130235	0.131493	0.132733	0.133957	0.135170	0.136371	0.137565	0.138753	0.139934	0.141112	0.142287
	$w2$	0.252494	0.255135	0.257717	0.260246	0.26273	0.265177	0.267592	0.269982	0.272350	0.274701	0.277038	0.279362	0.281676	0.283983
	$w3$	0.378377	0.382262	0.386055	0.389767	0.393408	0.396990	0.400523	0.404015	0.407471	0.410898	0.414299	0.417678	0.421035	0.424374
	$w4$	0.503742	0.508761	0.513652	0.518428	0.523105	0.527698	0.532219	0.536678	0.541083	0.545439	0.549749	0.554014	0.558233	0.562407
	$w5$	0.628226	0.634192	0.639988	0.645629	0.651134	0.656523	0.661806	0.666994	0.672089	0.677093	0.682004	0.686819	0.691531	0.696137
	$w6$	0.751123	0.757702	0.764051	0.770189	0.776133	0.781892	0.787469	0.792863	0.798068	0.803081	0.807896	0.812511	0.816924	0.821137
	$w7$	0.870634	0.877071	0.883135	0.888814	0.894106	0.899015	0.903551	0.907731	0.911578	0.915114	0.918365	0.921354	0.924105	0.926640
	$w8$	0.976742	0.979487	0.981672	0.983431	0.984867	0.986058	0.987059	0.987910	0.988641	0.989276	0.989831	0.990321	0.990755	0.991143
18	q	17.0505	14.0563	11.9049	10.2845	9.02068	8.00810	7.17917	6.48851	5.904520	5.40453	4.97186	4.59395	4.26119	3.966050
	$w1$	0.0594686	0.0600713	0.0606606	0.0612382	0.0618055	0.0623644	0.0629162	0.0634622	0.0640034	0.0645405	0.0650745	0.0656057	0.0661350	0.0666625
	$w2$	0.178353	0.180150	0.181907	0.183628	0.185318	0.186982	0.188625	0.190249	0.191859	0.193457	0.195044	0.196623	0.198196	0.199762
	$w3$	0.297073	0.300030	0.302917	0.305745	0.308519	0.311249	0.313942	0.316604	0.319240	0.321853	0.324449	0.327028	0.329595	0.332150
	$w4$	0.415486	0.419538	0.423490	0.427355	0.431143	0.434865	0.438533	0.442155	0.445737	0.449285	0.452802	0.456293	0.459760	0.463203
	$w5$	0.533390	0.538430	0.543337	0.548124	0.552807	0.557401	0.561919	0.566371	0.570764	0.575103	0.579394	0.583635	0.587829	0.591973
	$w6$	0.650449	0.656300	0.661976	0.667496	0.672878	0.678139	0.683293	0.688349	0.693310	0.698178	0.702952	0.707629	0.712205	0.716675
	$w7$	0.766004	0.772349	0.778464	0.784371	0.790085	0.795615	0.800967	0.806139	0.811128	0.815929	0.820540	0.824957	0.829181	0.833212
	$w8$	0.878366	0.884498	0.890267	0.895667	0.900695	0.905357	0.909663	0.913631	0.917283	0.920639	0.923724	0.926562	0.929174	0.931581
	$w9$	0.978128	0.980722	0.982786	0.984448	0.985805	0.986931	0.987876	0.988680	0.989372	0.989971	0.990497	0.990960	0.991371	0.991738
19	q	18.0825	14.9186	12.6451	10.9323	9.59615	8.52543	7.64875	6.91812	6.30025	5.77112	5.31317	4.91307	4.56070	4.24811
	$w1$	0.112345	0.113435	0.114501	0.115544	0.116568	0.117576	0.118571	0.119555	0.120529	0.121496	0.122456	0.123411	0.124361	0.125308
	$w2$	0.224605	0.226767	0.228880	0.230947	0.232976	0.234972	0.236941	0.238887	0.240815	0.242725	0.244623	0.246508	0.248385	0.250253
	$w3$	0.336682	0.339878	0.342998	0.346048	0.349040	0.351982	0.354881	0.357744	0.360578	0.363384	0.366169	0.368935	0.371684	0.374419
	$w4$	0.448450	0.452615	0.456675	0.460640	0.464523	0.468337	0.472091	0.475794	0.479454	0.483075	0.486663	0.490220	0.493749	0.497253
	$w5$	0.559722	0.564752	0.569645	0.574413	0.579074	0.583642	0.588131	0.592549	0.596906	0.601205	0.605452	0.609648	0.613793	0.617887
	$w6$	0.670184	0.675910	0.681460	0.686851	0.692103	0.697232	0.702253	0.707171	0.711996	0.716725	0.721361	0.725899	0.730337	0.734671
	$w7$	0.779218	0.785339	0.791235	0.796923	0.802420	0.807736	0.812877	0.817841	0.822627	0.827230	0.831650	0.835883	0.839930	0.843793
	$w8$	0.885232	0.891084	0.896586	0.901732	0.906521	0.910959	0.915057	0.918833	0.922306	0.925499	0.928435	0.931134	0.933620	0.935911
	$w9$	0.979360	0.981818	0.983774	0.985349	0.986635	0.987701	0.988598	0.989360	0.990015	0.990584	0.991082	0.991521	0.991911	0.992260
20	q	19.1156	15.7820	13.3862	11.5810	10.1725	9.04367	8.11921	7.34868	6.69689	6.13865	5.65540	5.23312	4.86115	4.53111
	$w1$	0.0532303	0.0537274	0.0542134	0.0546890	0.0551559	0.0556153	0.0560685	0.0565165	0.0569600	0.0573997	0.0578364	0.0582704	0.0587022	0.0591322
	$w2$	0.159657	0.161141	0.162592	0.164011	0.165404	0.166774	0.168126	0.169462	0.170783	0.172094	0.173395	0.174688	0.175973	0.177254
	$w3$	0.265977	0.268427	0.270819	0.273159	0.275454	0.277710	0.279934	0.282132	0.284305	0.286459	0.288596	0.290718	0.292829	0.294929
	$w4$	0.372106	0.375480	0.378773	0.381990	0.385143	0.388240	0.391291	0.394303	0.397279	0.400227	0.403149	0.406048	0.408928	0.411791
	$w5$	0.477926	0.482161	0.486288	0.490315	0.494255	0.498122	0.501925	0.505675	0.509376	0.513037	0.516661	0.520250	0.523810	0.527340
	$w6$	0.583265	0.58826	0.593117	0.597846	0.602465	0.606988	0.611428	0.615796	0.620098	0.624342	0.628530	0.632664	0.636747	0.640776
	$w7$	0.687828	0.693422	0.698842	0.704101	0.709220	0.714215	0.719100	0.723884	0.728571	0.733163	0.737661	0.742062	0.746363	0.750563
	$w8$	0.791030	0.796939	0.802627	0.808108	0.813401	0.818517	0.823460	0.828230	0.832826	0.837246	0.841487	0.845548	0.849431	0.853137
	$w9$	0.891370	0.896966	0.902224	0.907138	0.911708	0.915942	0.919851	0.923452	0.926763	0.929807	0.932606	0.935180	0.937550	0.939735
	$w10$	0.980461	0.982797	0.984656	0.986152	0.987374	0.988387	0.989239	0.989963	0.990586	0.991127	0.991600	0.992018	0.992389	0.992721

Table II. Prototype values of q and $w_r = \omega_{zr}/\omega_h$ for absorptive bandstop filters of order n with equal resonator Q_u and equiripple stopband attenuation $L_s = L_h$.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
21	q	20.1494	16.6461	14.1280	12.2304	10.7497	9.56273	8.59052	7.78006	7.09438	6.50702	5.99846	5.55401	5.16243	4.81502
	$w1$	0.101143	0.102051	0.102938	0.103807	0.104658	0.105496	0.106322	0.107139	0.107946	0.108747	0.109541	0.110330	0.111114	0.111896
	$w2$	0.202229	0.204034	0.205796	0.207521	0.209212	0.210874	0.212513	0.214132	0.215733	0.217319	0.218893	0.220456	0.222009	0.223557
	$w3$	0.303197	0.305873	0.308485	0.311039	0.313542	0.316002	0.318425	0.320818	0.323182	0.325524	0.327846	0.330151	0.332441	0.334721
	$w4$	0.403967	0.407474	0.410894	0.414234	0.417505	0.420716	0.423877	0.426995	0.430074	0.433121	0.436139	0.439132	0.442104	0.445059
	$w5$	0.504434	0.508709	0.512871	0.516930	0.520899	0.524792	0.528618	0.532387	0.536104	0.539778	0.543412	0.547010	0.550574	0.554112
	$w6$	0.604434	0.609380	0.614184	0.618860	0.623422	0.627887	0.632267	0.636573	0.640810	0.644986	0.649105	0.653169	0.657178	0.661138
	$w7$	0.703690	0.709153	0.714440	0.719568	0.724554	0.729417	0.734169	0.738819	0.743370	0.747828	0.752191	0.756458	0.760627	0.764699
	$w8$	0.801650	0.807359	0.812849	0.818137	0.823239	0.828167	0.832924	0.837514	0.841934	0.846182	0.850257	0.854158	0.857887	0.861448
	$w9$	0.896889	0.902250	0.907283	0.911985	0.916356	0.920404	0.924139	0.927580	0.930743	0.933652	0.936325	0.938784	0.941048	0.943137
	$w10$	0.981451	0.983677	0.985447	0.986873	0.988037	0.989002	0.989813	0.990503	0.991096	0.991611	0.992063	0.992461	0.992814	0.993131
22	q	21.1796	17.5110	14.8707	12.8807	11.3276	10.0826	9.06258	8.21217	7.49262	6.87614	6.34228	5.87567	5.46449	5.09956
	$w1$	0.0481642	0.0485883	0.0489964	0.0493957	0.0497873	0.0501726	0.0505521	0.0509271	0.051298	0.0516654	0.0520299	0.0523918	0.0527516	0.0531096
	$w2$	0.144470	0.145737	0.146957	0.148150	0.149319	0.150470	0.151603	0.152722	0.153829	0.154926	0.156013	0.157093	0.158166	0.159234
	$w3$	0.240704	0.242800	0.244816	0.246787	0.248718	0.250618	0.252488	0.254334	0.256160	0.257968	0.259760	0.261539	0.263306	0.265063
	$w4$	0.336813	0.339710	0.342494	0.345215	0.347881	0.350499	0.353077	0.355620	0.358132	0.360619	0.363083	0.365527	0.367954	0.370365
	$w5$	0.432726	0.436380	0.439891	0.443319	0.446673	0.449965	0.453203	0.456394	0.459545	0.462660	0.465744	0.468801	0.471833	0.474843
	$w6$	0.528345	0.532690	0.536865	0.540934	0.544910	0.548807	0.552635	0.556403	0.560118	0.563785	0.567411	0.570999	0.574551	0.578070
	$w7$	0.623518	0.628457	0.633198	0.637809	0.642306	0.646704	0.651015	0.655249	0.659415	0.663517	0.667559	0.671546	0.675478	0.679353
	$w8$	0.717983	0.723361	0.728517	0.733514	0.738370	0.743103	0.747723	0.752242	0.756663	0.760990	0.765222	0.769360	0.773401	0.777342
	$w9$	0.811214	0.816770	0.822074	0.827180	0.832102	0.836854	0.841438	0.845859	0.850114	0.854201	0.858122	0.861874	0.865460	0.868882
	$w10$	0.901856	0.907022	0.911849	0.916355	0.920543	0.924420	0.927997	0.931291	0.934319	0.937102	0.939661	0.942015	0.944182	0.946180
	$w11$	0.982342	0.984471	0.986162	0.987522	0.988634	0.989555	0.990329	0.990988	0.991554	0.992046	0.992477	0.992858	0.993196	0.993498
23	q	22.2192	18.3765	15.6140	13.5316	11.9063	10.6030	9.53531	8.64496	7.89152	7.24592	6.68677	6.19800	5.76722	5.38485
	$w1$	0.0919614	0.092731	0.0934826	0.0942179	0.0949386	0.0956472	0.0963452	0.0970343	0.0977158	0.0983906	0.0990597	0.0997241	0.100384	0.101040
	$w2$	0.183884	0.185415	0.186910	0.188372	0.189805	0.191213	0.192601	0.193970	0.195323	0.196663	0.197991	0.199310	0.200619	0.201921
	$w3$	0.275726	0.278002	0.280223	0.282394	0.284521	0.286611	0.288668	0.290697	0.292703	0.294688	0.296654	0.298606	0.300543	0.302469
	$w4$	0.367436	0.37043	0.373349	0.376200	0.378992	0.381732	0.384429	0.387087	0.389713	0.392309	0.394881	0.397431	0.399961	0.402474
	$w5$	0.458948	0.462619	0.466194	0.469683	0.473095	0.476441	0.479730	0.482970	0.486167	0.489327	0.492452	0.495549	0.498618	0.501663
	$w6$	0.550169	0.554457	0.558627	0.562689	0.566656	0.570541	0.574355	0.578107	0.581804	0.585452	0.589056	0.592621	0.596147	0.599639
	$w7$	0.640954	0.64577	0.650441	0.654981	0.659406	0.663731	0.667967	0.672126	0.676214	0.680238	0.684201	0.688107	0.691957	0.695750
	$w8$	0.731054	0.736255	0.741282	0.746151	0.750880	0.755486	0.759979	0.764371	0.768665	0.772865	0.776972	0.780985	0.784903	0.788723
	$w9$	0.819969	0.82531	0.830440	0.835374	0.840128	0.844714	0.849137	0.853399	0.857499	0.861438	0.865213	0.868827	0.872279	0.875573
	$w10$	0.906408	0.911352	0.915989	0.920316	0.924336	0.928055	0.931486	0.853399	0.937548	0.940217	0.942670	0.944926	0.947004	0.948920
	$w11$	0.983159	0.985193	0.986810	0.988111	0.989174	0.990056	0.990796	0.991426	0.991968	0.992439	0.992851	0.993215	0.993539	0.993828

Table II. Prototype values of q and $w_r = \omega_{zr}/\omega_h$ for absorptive bandstop filters of order n with equal resonator Q_u and equiripple stopband attenuation L_s=L_h.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
24	q	23.2550	19.2427	16.3579	14.1831	12.4855	11.1242	10.0087	9.07840	8.29107	7.61635	7.03192	6.52096	6.07059	5.67077
	$w1$	0.0439848	0.0443414	0.0446896	0.0450301	0.0453639	0.0456919	0.046015	0.0463339	0.0466492	0.0469613	0.0472707	0.0475777	0.0478827	0.0481859
	$w2$	0.131938	0.133005	0.134046	0.135064	0.136062	0.137042	0.138008	0.138961	0.139903	0.140835	0.141759	0.142676	0.143587	0.144492
	$w3$	0.219842	0.221608	0.223332	0.225017	0.226668	0.228289	0.229886	0.231461	0.233018	0.234558	0.236084	0.237598	0.239101	0.240595
	$w4$	0.307658	0.310106	0.312494	0.314827	0.317111	0.319354	0.321561	0.323738	0.325888	0.328015	0.330121	0.332209	0.334282	0.336341
	$w5$	0.395341	0.398443	0.401466	0.404417	0.407306	0.410139	0.412926	0.415672	0.418383	0.421063	0.423715	0.426343	0.42895	0.431538
	$w6$	0.482828	0.486544	0.490161	0.493688	0.497137	0.500517	0.503838	0.507107	0.510331	0.513516	0.516664	0.519781	0.522869	0.525932
	$w7$	0.570031	0.574302	0.578454	0.582495	0.586441	0.590303	0.594092	0.597817	0.601485	0.605103	0.608675	0.612205	0.615697	0.619151
	$w8$	0.656812	0.661553	0.666149	0.670614	0.674963	0.679211	0.683369	0.687449	0.691458	0.695401	0.699283	0.703106	0.706872	0.710582
	$w9$	0.742935	0.748009	0.752911	0.757655	0.762260	0.766742	0.771114	0.775383	0.779555	0.783634	0.787620	0.791514	0.795313	0.779016
	$w10$	0.827922	0.833095	0.838060	0.842832	0.847428	0.851859	0.856130	0.860244	0.864201	0.868000	0.871641	0.875124	0.878452	0.881626
	$w11$	0.910540	0.915300	0.919761	0.923922	0.927786	0.931360	0.934656	0.937690	0.940478	0.943041	0.945397	0.947564	0.949559	0.951399
	$w12$	0.983901	0.985850	0.987400	0.988648	0.989667	0.990511	0.991221	0.991825	0.992344	0.992795	0.993191	0.993540	0.99385	0.994127
25	q	24.2914	20.1094	17.1024	14.8352	13.0653	11.6458	10.4826	9.51239	8.69118	7.98734	7.37762	6.84450	6.37454	5.95736
	$w1$	0.0843007	0.0849622	0.0856079	0.0862392	0.0868579	0.0874657	0.0880643	0.0886548	0.0892385	0.089816	0.0903883	0.090956	0.0915198	0.0920807
	$w2$	0.168574	0.169891	0.171177	0.172434	0.173665	0.174874	0.176065	0.177240	0.178400	0.179549	0.180686	0.181815	0.182935	0.184050
	$w3$	0.252790	0.254752	0.256665	0.258535	0.260366	0.262164	0.263934	0.265679	0.267403	0.269108	0.270797	0.272472	0.274134	0.275787
	$w4$	0.336915	0.339502	0.342024	0.344487	0.346898	0.349264	0.351591	0.353886	0.356151	0.358390	0.360607	0.362804	0.364984	0.367151
	$w5$	0.420905	0.424089	0.427191	0.430218	0.433179	0.436083	0.438937	0.441749	0.444523	0.447263	0.449975	0.452660	0.455322	0.457967
	$w6$	0.504702	0.508445	0.512086	0.515636	0.519104	0.522501	0.525838	0.529121	0.532358	0.535552	0.538709	0.541833	0.544927	0.547997
	$w7$	0.588223	0.592467	0.596590	0.600602	0.604516	0.608345	0.612101	0.615791	0.619423	0.623002	0.626535	0.630025	0.633474	0.636890
	$w8$	0.671337	0.675999	0.680517	0.684904	0.689174	0.693343	0.697422	0.701422	0.705350	0.709211	0.713010	0.716751	0.720434	0.724064
	$w9$	0.753817	0.758767	0.763546	0.768170	0.772656	0.777019	0.781272	0.785423	0.789479	0.793441	0.797312	0.801091	0.804778	0.808374
	$w10$	0.835206	0.84022	0.845029	0.849650	0.854097	0.858382	0.862511	0.866486	0.870308	0.873976	0.877491	0.880853	0.884064	0.887129
	$w11$	0.914326	0.918914	0.923211	0.927218	0.930938	0.934377	0.937549	0.940467	0.943149	0.945614	0.947880	0.949964	0.951883	0.953653
	$w12$	0.984581	0.986452	0.987941	0.989138	0.990116	0.990927	0.991608	0.992188	0.992687	0.993120	0.993500	0.993835	0.994132	0.994399
26	q	25.3283	20.9767	17.8474	15.4878	13.6456	12.168	10.9570	9.94691	9.09182	8.35887	7.72386	7.16857	6.67902	6.24431
	$w1$	0.0404653	0.0407736	0.0410745	0.0413686	0.0416569	0.04194	0.0422188	0.0424938	0.0427654	0.0430342	0.0433005	0.0435646	0.0438268	0.0440872
	$w2$	0.121384	0.122307	0.123207	0.124087	0.124949	0.125796	0.126630	0.127452	0.128264	0.129068	0.129864	0.130653	0.131437	0.132215
	$w3$	0.202267	0.203797	0.205289	0.206747	0.208176	0.209578	0.210959	0.212320	0.213664	0.214994	0.216311	0.217616	0.218912	0.220199
	$w4$	0.283088	0.285212	0.287284	0.289306	0.291287	0.293231	0.295144	0.297029	0.298891	0.300732	0.302554	0.304360	0.306152	0.307931
	$w5$	0.363816	0.366515	0.369145	0.371712	0.374224	0.376689	0.379113	0.381500	0.383857	0.386185	0.388490	0.390773	0.393036	0.395282
	$w6$	0.444408	0.447655	0.450816	0.453898	0.456913	0.459868	0.462772	0.465630	0.468449	0.471233	0.473986	0.476711	0.479411	0.482089
	$w7$	0.524812	0.528567	0.532219	0.535777	0.539252	0.542655	0.545995	0.549280	0.552516	0.555709	0.558863	0.561983	0.565071	0.568129
	$w8$	0.604947	0.609156	0.613243	0.617217	0.621093	0.624883	0.628599	0.632247	0.635836	0.639373	0.642861	0.646305	0.649707	0.653070
	$w9$	0.684689	0.689271	0.693709	0.698016	0.702207	0.706295	0.710295	0.714214	0.718061	0.721840	0.725558	0.729216	0.732817	0.736360
	$w10$	0.763819	0.768650	0.773311	0.777818	0.782188	0.786437	0.790577	0.794615	0.798558	0.802410	0.806170	0.809841	0.813420	0.816907
	$w11$	0.841902	0.846766	0.851428	0.855905	0.860212	0.864361	0.868356	0.872201	0.875896	0.879442	0.882838	0.886087	0.889189	0.892147
	$w12$	0.917805	0.922233	0.926379	0.930243	0.933828	0.937143	0.940199	0.943010	0.945594	0.947968	0.950149	0.952156	0.954004	0.955709
	$w13$	0.985206	0.987006	0.988437	0.989588	0.990529	0.991308	0.991963	0.992521	0.993000	0.993417	0.993782	0.994104	0.994390	0.994647

Table II. Prototype values of q and $w_r = \omega_{zr}/\omega_h$ for absorptive bandstop filters of order n with equal resonator Q_u and equiripple stopband attenuation L_s=L_h.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
2	J1	1.20185	1.11282	1.06331	1.03558	1.02000	1.01125	1.00632	1.00356	1.00200	1.00112	1.00063	1.00036	1.00020	1.00011
3	J1	2.06817	1.90660	1.80908	1.74823	1.70931	1.68397	1.66725	1.65611	1.64864	1.64361	1.64020	1.63789	1.63633	1.63526
	J2	1.06708	0.904196	0.797743	0.726744	0.678877	0.646438	0.624391	0.609389	0.599175	0.592218	0.587479	0.584251	0.582052	0.580553
4	J1	2.91730	2.68593	2.54225	2.44923	2.38710	2.34454	2.31478	2.29364	2.27843	2.26739	2.25930	2.25335	2.24895	2.24568
	J2	1.72990	1.47760	1.30980	1.19442	1.11329	1.05528	1.01326	0.982561	0.95998	0.943286	0.930901	0.921686	0.914817	0.909689
	J3	1.23210	1.01535	0.864552	0.756361	0.677366	0.619077	0.575764	0.543462	0.519323	0.501257	0.487727	0.477587	0.469988	0.464290
5	J1	3.76089	3.46061	3.27147	3.14689	3.06195	3.00242	2.95973	2.92853	2.90538	2.88797	2.87474	2.86461	2.85678	2.85071
	J2	2.35882	2.02368	1.79877	1.64202	1.52974	1.44766	1.38668	1.34076	1.30583	1.27904	1.25835	1.24228	1.22975	1.21993
	J3	1.90270	1.58052	1.35678	1.19566	1.07671	0.987334	0.919253	0.866852	0.826204	0.794488	0.769637	0.750098	0.734698	0.722536
	J4	1.42554	1.17064	0.988609	0.853565	0.750921	0.671733	0.610018	0.561603	0.523462	0.493333	0.469494	0.450607	0.435631	0.423749
6	J1	4.60206	4.23324	3.99888	3.84299	3.73550	3.65919	3.60369	3.56252	3.53146	3.50769	3.48927	3.47486	3.46348	3.45443
	J2	2.97569	2.55999	2.27934	2.08223	1.93969	1.83424	1.75481	1.69412	1.64716	1.61046	1.58153	1.55855	1.54019	1.52544
	J3	2.51546	2.10044	1.81199	1.60361	1.44883	1.33141	1.24085	1.17012	1.11430	1.06989	1.03431	1.00565	0.982464	0.963637
	J4	2.14173	1.76649	1.49999	1.30324	1.15403	1.03863	0.948058	0.876185	0.818637	0.772238	0.734630	0.704009	0.679004	0.658529
	J5	1.60892	1.32469	1.11956	0.965096	0.845324	0.750627	0.674732	0.613364	0.563431	0.522631	0.489202	0.461749	0.439178	0.420600
7	J1	5.44199	5.00473	4.72539	4.53829	4.40831	4.31532	4.24711	4.19605	4.15713	4.12702	4.10343	4.08475	4.06980	4.05774
	J2	3.58691	3.09155	2.75594	2.51893	2.34644	2.21795	2.12039	2.04516	1.98638	1.93993	1.90288	1.87309	1.84895	1.82927
	J3	3.10626	2.60310	2.25314	1.99954	1.81034	1.66599	1.55388	1.46558	1.39522	1.33861	1.29271	1.25526	1.22452	1.19915
	J4	2.77384	2.29594	1.95760	1.70811	1.51880	1.37209	1.25643	1.16402	1.08939	1.02856	0.978657	0.937461	0.903302	0.874842
	J5	2.38199	1.96471	1.66504	1.44056	1.26741	1.13111	1.02208	0.933806	0.861665	0.802229	0.752988	0.711984	0.677710	0.648950
	J6	1.77629	1.46737	1.24377	1.07435	0.941713	0.835527	0.749071	0.677856	0.618707	0.569277	0.527818	0.492939	0.463538	0.438704
8	J1	6.28108	5.77561	5.45131	5.23306	5.08071	4.97109	4.89021	4.82928	4.78254	4.74614	4.71741	4.69446	4.67595	4.66089
	J2	4.19482	3.62056	3.23024	2.95358	2.75141	2.60005	2.48450	2.39486	2.32438	2.26831	2.22325	2.18671	2.15685	2.13229
	J3	3.68606	3.09731	2.68722	2.38939	2.16651	1.99578	1.86253	1.75702	1.67244	1.60394	1.54799	1.50194	1.46382	1.43207
	J4	3.37169	2.79882	2.39369	2.09502	1.86825	1.69209	1.55273	1.44091	1.35011	1.27567	1.21413	1.16291	1.12005	1.08400
	J5	3.04654	2.51758	2.13891	1.85611	1.63862	1.46760	1.33077	1.21982	1.12883	1.05352	0.990725	0.938009	0.893542	0.855858
	J6	2.60796	2.15553	1.82895	1.58239	1.39041	1.23747	1.11353	1.01181	0.927469	0.856963	0.797637	0.747435	0.704781	0.668400
	J7	1.92905	1.59795	1.35828	1.17634	1.03347	0.918374	0.823879	0.745209	0.679025	0.622920	0.575099	0.534166	0.499040	0.468830
9	J1	7.11965	6.54592	6.17681	5.92754	5.75286	5.62662	5.53311	5.46234	5.40779	5.36512	5.33125	5.30405	5.28199	5.26393
	J2	4.80079	4.14778	3.70308	3.38702	3.15531	2.98116	2.84772	2.74376	2.66164	2.59603	2.54300	2.49977	2.46423	2.43481
	J3	4.25967	3.58648	3.11721	2.77578	2.51964	2.32278	2.16867	2.04615	1.94751	1.86730	1.80143	1.74692	1.70152	1.66348
	J4	3.95191	3.28792	2.81876	2.47282	2.20985	2.00512	1.84278	1.71210	1.60558	1.51792	1.44509	1.38415	1.33285	1.28944
	J5	3.66429	3.03334	2.58278	2.24689	1.98885	1.78591	1.62346	1.49149	1.38297	1.29291	1.21746	1.15381	1.09981	1.05378
	J6	3.30990	2.73780	2.32596	2.01595	1.77533	1.58414	1.42957	1.30286	1.19778	1.10988	1.03570	0.972711	0.918919	0.872780
	J7	2.81723	2.33368	1.98418	1.71940	1.51215	1.34582	1.20993	1.09731	1.00295	0.923241	0.855384	0.797293	0.747328	0.704195
	J8	2.06977	1.71799	1.46365	1.27058	1.11888	0.996373	0.895437	0.810918	0.739293	0.678040	0.625267	0.579564	0.539830	0.505204
10	J1	7.95790	7.31605	6.90205	6.62179	6.42477	6.28198	6.17587	6.09527	6.03294	5.98400	5.94501	5.91357	5.88797	5.86692
	J2	5.40548	4.67407	4.17500	3.81963	3.5584	3.36157	3.21034	3.09211	2.99842	2.92328	2.86233	2.81244	2.77126	2.73702
	J3	4.82945	4.07277	3.54466	3.15996	2.87073	2.64798	2.47318	2.33378	2.22122	2.12937	2.05368	1.99079	1.93821	1.89395
	J4	4.52186	3.76926	3.23746	2.84525	2.54673	2.31400	2.12912	1.97989	1.85794	1.75725	1.67333	1.60284	1.54328	1.49265
	J5	4.25700	3.52969	3.01091	2.62467	2.32798	2.09463	1.90770	1.75556	1.63022	1.52590	1.43827	1.36409	1.30093	1.24684
	J6	3.95361	3.27331	2.78440	2.41729	2.13284	1.90722	1.72503	1.57565	1.45172	1.34790	1.26012	1.18537	1.12135	1.06621
	J7	3.55764	2.94779	2.50747	2.17469	1.91482	1.70694	1.53762	1.39762	1.28055	1.18176	1.09768	1.02563	0.963568	0.909825
	J8	3.01144	2.49938	2.12914	1.84847	1.6282	1.45083	1.30519	1.18371	1.08120	0.993908	0.918966	0.854230	0.798051	0.749080
	J9	2.20085	1.82957	1.56127	1.35787	1.19805	1.06901	0.962556	0.873196	0.797176	0.731842	0.675181	0.625724	0.582365	0.544198

Table III. Normalized admittance inverter values, $J_r = J_r/g$, for equal-resonator-Q_u absorptive bandstop filters and equiripple stopband attenuation L_S (J_0 is given by (10)).

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
11	J1	8.79586	8.08593	7.62717	7.31588	7.09655	6.93723	6.81851	6.72811	6.65801	6.60279	6.55869	6.52303	6.49389	6.46985
	J2	6.00925	5.19956	4.64634	4.25165	3.96098	3.74154	3.57251	3.44008	3.33484	3.25019	3.18136	3.12485	3.07805	3.03900
	J3	5.39663	4.55697	3.97049	3.54264	3.2205	2.97200	2.77656	2.62040	2.49399	2.39055	2.30510	2.23393	2.17420	2.12376
	J4	5.08527	4.24553	3.65218	3.21426	2.88065	2.62025	2.41302	2.24547	2.10824	1.99466	1.89978	1.81989	1.75216	1.69440
	J5	4.83459	4.01430	3.42978	2.99472	2.66058	2.39768	2.18683	2.01503	1.87325	1.75500	1.65546	1.57102	1.49889	1.43694
	J6	4.56492	3.78301	3.22197	2.80126	2.47571	2.21774	2.00944	1.83864	1.69682	1.57784	1.47712	1.39118	1.31738	1.25363
	J7	4.22972	3.50605	2.98428	2.59057	2.28378	2.03890	1.83976	1.67537	1.53801	1.42209	1.32342	1.23880	1.16577	1.10239
	J8	3.78943	3.14515	2.67961	2.32701	2.05080	1.82892	1.64720	1.49607	1.36887	1.26078	1.16816	1.08826	1.01891	0.958415
	J9	3.19288	2.65395	2.26450	1.96924	1.73740	1.55046	1.39655	1.26772	1.15849	1.06495	0.984162	0.913925	0.852516	0.798583
	J10	2.32404	1.93415	1.65262	1.4393	1.27183	1.13670	1.02522	0.931607	0.851843	0.783103	0.723286	0.670845	0.624587	0.583595
12	J1	9.63369	8.85568	8.35212	8.00986	7.76824	7.59241	7.46105	7.36087	7.28301	7.22155	7.17234	7.13245	7.09977	7.06949
	J2	6.61241	5.72454	5.11717	4.68327	4.36320	4.12120	3.93435	3.78773	3.67099	3.57688	3.50019	3.43706	3.38464	3.33544
	J3	5.96207	5.03978	4.39507	3.92429	3.56935	3.29520	3.07914	2.90626	2.76608	2.65113	2.55597	2.47652	2.40967	2.34614
	J4	5.64434	4.71846	4.06413	3.58096	3.21256	2.92472	2.69524	2.50949	2.35713	2.23078	2.12502	2.03578	1.95994	1.88718
	J5	5.40244	4.49140	3.84260	3.35983	2.98895	2.69705	2.46262	2.27147	2.11352	1.98157	1.87031	1.77572	1.69475	1.61642
	J6	5.15619	4.27693	3.64677	3.17474	2.80975	2.52068	2.28714	2.09564	1.93653	1.80288	1.68959	1.59276	1.50943	1.42795
	J7	4.86279	4.03273	3.43497	2.98461	2.63419	2.35493	2.12796	1.94079	1.78446	1.65250	1.54011	1.44361	1.36021	1.27768
	J8	4.49038	3.72737	3.17642	2.75966	2.43370	2.17239	1.95869	1.78138	1.63240	1.50594	1.39767	1.30427	1.22318	1.14239
	J9	4.00700	3.33046	2.84163	2.47122	2.18062	1.94669	1.75435	1.59381	1.45807	1.34211	1.24221	1.15551	1.07982	1.00492
	J10	3.36378	2.79919	2.39144	2.08248	1.83993	1.64434	1.48305	1.34786	1.23297	1.13421	1.04858	0.973749	0.907965	0.844578
	J11	2.44074	2.03298	1.73871	1.51590	1.34108	1.20016	1.08390	0.98633	0.903202	0.831489	0.768987	0.714047	0.665412	0.620574
13	J1	10.4714	9.62527	9.07697	8.70375	8.43986	8.24746	8.10357	7.99359	7.90798	7.84025	7.78594	7.74184	7.70563	7.67563
	J2	7.21509	6.24907	5.58765	5.11457	4.76515	4.50051	4.29599	4.13520	4.00696	3.90339	3.81884	3.74912	3.69112	3.64251
	J3	6.5262	5.52151	4.81879	4.30516	3.91752	3.61767	3.38117	3.19162	3.03768	2.91124	2.80640	2.71871	2.64480	2.58211
	J4	6.20031	5.18897	4.47414	3.94602	3.54303	3.22779	2.97632	2.77246	2.60502	2.46595	2.34937	2.25085	2.16697	2.09514
	J5	5.96357	4.96335	4.25134	3.72155	3.31445	2.99378	2.73618	2.52587	2.35190	2.20636	2.08348	1.97887	1.88918	1.81183
	J6	5.73409	4.76040	4.06324	3.54138	3.13804	2.81854	2.56052	2.34877	2.17270	2.02465	1.89902	1.79152	1.69888	1.61857
	J7	5.47102	4.53948	3.86939	3.36513	2.97321	2.66107	2.40768	2.19872	2.02419	1.87679	1.75118	1.64325	1.54987	1.46860
	J8	5.14638	4.27273	3.64243	3.16616	2.79418	2.49634	2.25326	2.05175	1.88260	1.73909	1.61626	1.51031	1.41828	1.33789
	J9	4.73621	3.93653	3.35897	2.92156	2.57878	2.30312	2.07702	1.88858	1.72955	1.59391	1.47724	1.37611	1.28788	1.21050
	J10	4.21233	3.50506	2.99430	2.60730	2.30360	2.05878	1.85729	1.68863	1.54556	1.42288	1.31676	1.22426	1.14312	1.07159
	J11	3.52580	2.93656	2.51128	2.18920	1.93649	1.73269	1.56473	1.42383	1.30392	1.20065	1.11088	1.03219	0.962764	0.901179
	J12	2.55188	2.12695	1.82043	1.58845	1.40654	1.25994	1.13917	1.03784	0.951532	0.877072	0.812156	0.755037	0.704390	0.659210
14	J1	11.3089	10.3948	9.80175	9.39755	9.11142	8.9025	8.74602	8.62627	8.53290	8.45894	8.39952	8.35119	8.31147	8.27849
	J2	7.81737	6.77333	6.05786	5.5456	5.16687	4.87968	4.65744	4.48250	4.34278	4.22978	4.13737	4.06106	3.99749	3.94410
	J3	7.08934	6.00250	5.24185	4.68543	4.26516	3.93972	3.68275	3.47657	3.30890	3.17102	3.05650	2.96058	2.87964	2.81085
	J4	6.75401	5.65778	4.88275	4.30984	3.87243	3.52996	3.25650	3.03461	2.85214	2.70043	2.57307	2.46527	2.37341	2.29459
	J5	6.51991	5.43167	4.65717	4.08083	3.63787	3.28874	3.00808	2.77877	2.58888	2.42988	2.29545	2.18087	2.08253	1.99757
	J6	6.30265	5.23669	4.47396	3.90328	3.46235	3.11304	2.83085	2.59916	2.40638	2.24417	2.10634	1.98829	1.88646	1.79805
	J7	6.06235	5.03285	4.29296	3.73664	3.30467	2.96081	2.68176	2.45167	2.25943	2.09703	1.95852	1.83942	1.73630	1.64643
	J8	5.77322	4.79441	4.08878	3.55609	3.14057	2.80823	2.53725	2.31281	2.12450	1.96480	1.82808	1.71011	1.60762	1.51800
	J9	5.41523	4.50120	3.84128	3.34184	2.95088	2.63687	2.37966	2.16562	1.98520	1.83151	1.69939	1.58494	1.48515	1.39759
	J10	4.96883	4.13443	3.53188	3.07541	2.71744	2.42911	2.19207	1.99399	1.82624	1.68267	1.55866	1.45074	1.35623	1.27297
	J11	4.40724	3.67053	3.13874	2.73594	2.41995	2.16516	1.95530	1.77944	1.62997	1.50151	1.39002	1.29252	1.20672	1.13075
	J12	3.68024	3.06732	2.62511	2.29035	2.02787	1.81626	1.64193	1.49570	1.37120	1.26393	1.17051	1.08847	1.01594	0.951384
	J13	2.65820	2.21678	1.89843	1.65755	1.46880	1.31673	1.19153	1.08655	0.997185	0.920135	0.852951	0.793828	0.741391	0.694541

Table III. Normalized admittance inverter values, $J_r = J_r/g$, for equal-resonator-Q_u absorptive bandstop filters and equiripple stopband attenuation L_S (J_0 is given by (10)).

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
15	J1	12.1464	11.1643	10.5265	10.0913	9.78291	9.5575	9.38845	9.25891	9.1578	9.07759	9.01307	8.96053	8.91728	8.88132
	J2	8.41937	7.29736	6.52787	5.97648	5.5684	5.2587	5.01875	4.82967	4.67848	4.55605	4.45581	4.37292	4.30376	4.24561
	J3	7.65175	6.48291	5.66441	5.0653	4.61237	4.26141	3.984	3.7612	3.57984	3.43052	3.30636	3.20223	3.11424	3.03937
	J4	7.30604	6.12531	5.29029	4.67279	4.20099	3.8314	3.53602	3.29614	3.0987	2.93436	2.79626	2.67925	2.57938	2.49362
	J5	7.07274	5.89735	5.06087	4.43837	3.95973	3.58235	3.27876	3.03054	2.82484	2.65243	2.50653	2.38204	2.27506	2.18256
	J6	6.86447	5.70782	4.88057	4.26184	3.78377	3.40507	3.09899	2.84758	2.63827	2.462	2.31213	2.18363	2.07266	1.97624
	J7	6.64166	5.51679	4.70892	4.10195	3.63086	3.25607	2.95192	2.70113	2.49154	2.31441	2.16324	2.03317	1.92042	1.8221
	J8	6.3799	5.29984	4.52174	3.93488	3.47747	3.11201	2.8142	2.56767	2.36089	2.18552	2.03536	1.90575	1.79306	1.69449
	J9	6.06131	5.03879	4.30085	3.74277	3.30628	2.95615	2.66963	2.43147	2.23091	2.06017	1.91347	1.78641	1.6756	1.57839
	J10	5.67049	4.71829	4.03074	3.51009	3.10198	2.77364	2.50398	2.27895	2.08864	1.92597	1.78563	1.66362	1.55683	1.46283
	J11	5.19005	4.32237	3.69597	3.22154	2.84938	2.5495	2.30265	2.09602	1.92065	1.77014	1.63975	1.52591	1.42585	1.33741
	J12	4.59328	3.82818	3.2761	2.85813	2.53031	2.26611	2.04845	1.86598	1.71077	1.57716	1.46099	1.35917	1.26929	1.18949
	J13	3.8282	3.19241	2.73382	2.38681	2.11478	1.89566	1.71518	1.56386	1.43506	1.32404	1.22733	1.14232	1.06702	0.999904
	J14	2.76035	2.30299	1.97321	1.72375	1.52829	1.37093	1.24141	1.13288	1.04055	0.960968	0.891619	0.830609	0.776479	0.728121
16	J1	12.9839	11.9328	11.2505	10.7846	10.4544	10.2122	10.0308	9.89154	9.78268	9.69623	9.62661	9.56987	9.52308	9.48416
	J2	9.02122	7.82024	6.99705	6.40663	5.96982	5.63721	5.37996	5.17675	5.0141	4.88225	4.77417	4.68472	4.60998	4.54709
	J3	8.21365	6.96187	6.08588	5.44418	4.95929	4.58238	4.28499	4.0456	3.85055	3.68981	3.55602	3.44372	3.34868	3.26776
	J4	7.85684	6.59083	5.6963	5.03438	4.52894	4.13183	3.81503	3.5572	3.34481	3.16788	3.01906	2.89288	2.78503	2.69234
	J5	7.62299	6.35999	5.46225	4.79391	4.28044	3.87447	3.54851	3.28147	3.06	2.87424	2.71692	2.58259	2.467	2.36699
	J6	7.42132	6.17407	5.28341	4.61724	4.10311	3.6948	3.36551	3.09454	2.86881	2.6786	2.51676	2.37791	2.25786	2.15349
	J7	7.2122	5.99279	5.11862	4.46206	3.95327	3.54761	3.21925	2.94808	2.72139	2.52971	2.36604	2.22515	2.1029	1.99624
	J8	6.97207	5.79253	4.9444	4.30507	3.80773	3.40962	3.08621	2.81819	2.59339	2.40273	2.23944	2.09847	1.97579	1.86846
	J9	6.68431	5.55639	4.74387	4.12967	3.6503	3.26511	2.95106	2.68981	2.46993	2.28283	2.12211	1.98294	1.8615	1.75498
	J10	6.3358	5.27093	4.5033	3.9221	3.46746	3.10097	2.80118	2.55086	2.3394	2.15881	2.00314	1.86792	1.74956	1.64544
	J11	5.91373	4.92383	4.21036	3.66989	3.24668	2.90481	2.62453	2.38974	2.19071	2.0201	1.8725	1.74381	1.63078	1.53101
	J12	5.40149	4.50052	3.85148	3.3599	2.97499	2.66378	2.40839	2.19401	2.01181	1.85516	1.71916	1.60015	1.49521	1.40224
	J13	4.77169	3.97804	3.40662	2.97402	2.63546	2.36171	2.13711	1.94842	1.78787	1.64958	1.52922	1.42358	1.33011	1.247
	J14	3.97046	3.31159	2.83742	2.47858	2.19792	1.97103	1.78501	1.62877	1.49582	1.38125	1.28144	1.1937	1.1159	1.04651
	J15	2.8588	2.38531	2.04466	1.78692	1.58543	1.42258	1.28917	1.17717	1.08192	0.999884	0.928426	0.865603	0.809852	0.76008
17	J1	13.8212	12.703	11.9758	11.4787	11.1258	10.8674	10.6732	10.5241	10.4075	10.3148	10.2401	10.1792	10.1289	10.087
	J2	9.6228	8.34486	7.46741	6.8378	6.37114	6.01638	5.74107	5.52374	5.34962	5.20836	5.09246	4.99643	4.91614	4.84848
	J3	8.77499	7.44238	6.5084	5.824	5.30596	4.90396	4.58575	4.32978	4.12106	3.94892	3.80552	3.68502	3.58298	3.49598
	J4	8.40653	7.05752	6.10306	5.39666	4.85641	4.43268	4.09361	3.81786	3.59055	3.40106	3.24155	3.10615	2.99039	2.89077
	J5	8.17109	6.82304	5.86378	5.14967	4.60025	4.16667	3.81752	3.53171	3.29453	3.09547	2.92675	2.78257	2.65844	2.55092
	J6	7.97423	6.63938	5.68535	4.97201	4.42083	3.98402	3.63077	3.34036	3.0983	2.89423	2.72047	2.57129	2.44225	2.32994
	J7	7.77597	6.46561	5.52562	4.82008	4.27286	3.83764	3.48445	3.19312	2.9495	2.74342	2.56738	2.41572	2.2841	2.16914
	J8	7.55319	6.27857	5.36148	4.67075	4.1331	3.70399	3.35461	3.06551	2.82304	2.61736	2.44116	2.28895	2.15649	2.04049
	J9	7.29048	6.06249	5.17712	4.50844	3.98628	3.5681	3.22645	2.94284	2.70423	2.50125	2.32689	2.17588	2.04414	1.92849
	J10	6.97482	5.80407	4.95913	4.31988	3.81949	3.41756	3.08813	2.81373	2.58212	2.38445	2.21416	2.06626	1.93688	1.82302
	J11	6.5981	5.49492	4.69857	4.09561	3.62296	3.24252	2.92988	2.66869	2.4475	2.25811	2.09441	1.95178	1.82665	1.71622
	J12	6.1464	5.12232	4.38329	3.82374	3.38495	3.03144	2.74047	2.49687	2.29003	2.11239	1.95835	1.82369	1.70516	1.6002
	J13	5.60402	4.67305	4.00148	3.49321	3.09471	2.77365	2.50927	2.2877	2.09926	1.93709	1.79608	1.67245	1.56326	1.46626
	J14	4.94354	4.12431	3.53351	3.08653	2.73622	2.45409	2.22182	2.02715	1.86152	1.71884	1.5946	1.48545	1.38881	1.30271
	J15	4.10752	3.42817	2.93835	2.5679	2.27768	2.04406	1.85182	1.69077	1.5538	1.43581	1.33304	1.24268	1.16258	1.09109
	J16	2.95391	2.46617	2.11457	1.8487	1.64048	1.47291	1.33508	1.21969	1.12158	1.03713	0.963605	0.898976	0.841683	0.790523

Table III. Normalized admittance inverter values, $J_r = J_r/g$, for equal-resonator-Q_u absorptive bandstop filters and equiripple stopband attenuation L_S (J_0 is given by (10)).

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
18	J1	14.6585	13.4724	12.7003	12.1724	11.7972	11.5223	11.3156	11.1567	11.0324	10.9335	10.8537	10.7885	10.7347	10.6898
	J2	10.2243	8.86844	7.93696	7.26829	6.77235	6.39507	6.1021	5.87065	5.6851	5.53443	5.41071	5.30811	5.22226	5.14984
	J3	9.33599	7.92167	6.92994	6.20297	5.6524	5.22491	4.88634	4.6138	4.39143	4.20789	4.05488	3.92621	3.81717	3.72411
	J4	8.95544	7.52263	6.50855	5.75785	5.18345	4.73272	4.37185	4.07821	3.83601	3.63397	3.46378	3.31922	3.19552	3.08899
	J5	8.71759	7.28391	6.26357	5.50395	4.91931	4.45777	4.08594	3.78141	3.52857	3.31623	3.13615	2.98216	2.8495	2.73449
	J6	8.52417	7.10145	6.08473	5.32465	4.73728	4.27167	3.89503	3.58527	3.32698	3.10909	2.92349	2.76403	2.62602	2.50581
	J7	8.33456	6.93347	5.9287	5.17486	4.59027	4.12533	3.748	3.43669	3.17629	2.95593	2.7676	2.60527	2.46431	2.34111
	J8	8.12604	6.75708	5.7725	5.03142	4.45483	3.99478	3.6203	3.31044	3.05055	2.83004	2.6411	2.47781	2.33566	2.21108
	J9	7.88339	6.55683	5.60074	4.87916	4.31603	3.8653	3.49727	3.19186	2.935	2.71649	2.52881	2.36622	2.22436	2.09976
	J10	7.59554	6.32118	5.40162	4.7063	4.16233	3.72569	3.36808	3.07043	2.81934	2.60515	2.4207	2.26052	2.12043	1.99711
	J11	7.25359	6.04092	5.16554	4.50301	3.98386	3.56625	3.22334	2.9371	2.69491	2.4877	2.30873	2.15289	2.01624	1.89566
	J12	6.84874	5.70764	4.88405	4.26059	3.77175	3.37804	3.05418	2.78322	2.55337	2.35614	2.18529	2.03608	1.90485	1.78873
	J13	6.36978	5.31159	4.54808	3.97026	3.5172	3.1522	2.85172	2.59999	2.38606	2.20207	2.04227	1.9023	1.77885	1.66927
	J14	5.79959	4.83844	4.14519	3.62072	3.20964	2.87852	2.60594	2.3775	2.1832	2.01588	1.8703	1.7425	1.62948	1.52889
	J15	5.1089	4.2639	3.65457	3.19374	2.83268	2.54197	2.30275	2.10231	1.93183	1.78498	1.65711	1.54473	1.44518	1.35639
	J16	4.24024	3.5401	3.03529	2.65362	2.35466	2.11404	1.91611	1.75036	1.60944	1.4881	1.38245	1.28958	1.20727	1.13378
	J17	3.04595	2.54373	2.18168	1.90798	1.69363	1.52116	1.37934	1.26062	1.15974	1.07291	0.997356	0.930972	0.872157	0.819643
19	J1	15.4958	14.2416	13.4249	12.866	12.4686	12.1772	11.9579	11.7893	11.6572	11.552	11.4672	11.3978	11.3404	11.2926
	J2	10.8255	9.39181	8.40647	7.69871	7.17348	6.77371	6.46308	6.21751	6.02051	5.86044	5.72891	5.61975	5.52834	5.45118
	J3	9.89664	8.4006	7.35131	6.58177	5.99868	5.54571	5.18678	4.89768	4.66167	4.46674	4.30414	4.1673	4.05125	3.95215
	J4	9.50364	7.98713	6.91367	6.11869	5.51017	5.03246	4.64983	4.3383	4.08123	3.86664	3.6858	3.53208	3.40047	3.28704
	J5	9.26276	7.74368	6.66262	5.85757	5.2378	4.74835	4.35389	4.03066	3.7622	3.5366	3.3452	3.18141	3.04024	2.91776
	J6	9.07168	7.56161	6.48272	5.67612	5.05272	4.55845	4.15852	3.82946	3.55499	3.32334	3.12593	2.95622	2.80928	2.68121
	J7	8.88903	7.39808	6.32933	5.52762	4.90598	4.41156	4.01028	3.6791	3.40202	3.16746	2.96692	2.79398	2.64374	2.51236
	J8	8.69222	7.23029	6.17941	5.38871	4.77372	4.28316	3.8839	3.55352	3.2764	3.04121	2.83965	2.66539	2.51362	2.38056
	J9	8.46646	7.04323	6.018	5.24457	4.64129	4.15867	3.76477	3.43796	3.16315	2.92938	2.72858	2.55459	2.40274	2.26933
	J10	8.20117	6.82585	5.83386	5.08404	4.49775	4.02745	3.64253	3.32228	3.05229	2.82203	2.62379	2.45165	2.30109	2.16855
	J11	7.88825	6.56967	5.61807	4.89799	4.33399	3.88058	3.50853	3.19817	2.93578	2.71142	2.51775	2.34917	2.2014	2.07103
	J12	7.52055	6.26751	5.36322	4.67868	4.14206	3.71005	3.3549	3.05794	2.80626	2.59046	2.40369	2.24068	2.09742	1.97073
	J13	7.08991	5.91202	5.06219	4.41895	3.91463	3.50839	3.17408	2.89412	2.65637	2.45206	2.27477	2.11963	1.98291	1.86168
	J14	6.585	5.49364	4.70648	4.11085	3.64398	3.26793	2.95838	2.69898	2.47846	2.28865	2.12362	1.97888	1.85101	1.73731
	J15	5.98814	4.99768	4.28348	3.74324	3.31992	2.97907	2.69855	2.4635	2.26361	2.09145	1.94161	1.80999	1.69349	1.58968
	J16	5.26908	4.39902	3.77175	3.29738	2.9258	2.62671	2.38066	2.17456	1.99935	1.84844	1.71708	1.60162	1.49932	1.40805
	J17	4.36869	3.64835	3.12909	2.73646	2.42898	2.18156	1.97808	1.80772	1.66295	1.53831	1.42985	1.33454	1.25009	1.1747
	J18	3.13526	2.61894	2.2468	1.96544	1.74512	1.56787	1.42215	1.30018	1.19657	1.10741	1.02987	0.961754	0.90143	0.847598

Table III. Normalized admittance inverter values, $\mathrm{J_r}=J_r/g$, for equal-resonator-Q_u absorptive bandstop filters and equiripple stopband attenuation L_S (J_0 is given by (10)).

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
20	J1	16.3331	15.0109	14.1495	13.5596	13.14	12.8321	12.6002	12.4219	12.282	12.1706	12.0806	12.007	11.9462	11.8954
	J2	11.4268	9.91519	8.8759	8.12905	7.57454	7.15228	6.82399	6.56433	6.35588	6.18641	6.04707	5.93135	5.83439	5.75248
	J3	10.4571	8.87941	7.7725	6.9604	6.34479	5.86638	5.48709	5.18146	4.9318	4.7255	4.55331	4.40831	4.28526	4.18012
	J4	10.0514	8.4513	7.31843	6.47922	5.8366	5.33195	4.92756	4.59818	4.32624	4.09914	3.90765	3.74479	3.60526	3.48494
	J5	9.80695	8.20275	7.06102	6.21064	5.55579	5.0385	4.62143	4.27957	3.99548	3.75667	3.55395	3.38039	3.23071	3.10079
	J6	9.61735	8.02041	6.87957	6.02664	5.36733	4.84452	4.42135	4.07309	3.78246	3.5371	3.3279	3.14798	2.99212	2.85621
	J7	9.44031	7.86028	6.72799	5.87877	5.22031	4.69662	4.2715	3.9206	3.62691	3.37822	3.16552	2.98202	2.82253	2.683
	J8	9.25328	7.69952	6.58307	5.74334	5.09038	4.56965	4.14583	3.79515	3.50093	3.25119	3.03709	2.85194	2.69062	2.54914
	J9	9.04172	7.52341	6.43012	5.60574	4.96299	4.44902	4.02963	3.68177	3.38925	3.14042	2.92666	2.74141	2.57969	2.43756
	J10	8.79539	7.32129	6.25835	5.45527	4.82764	4.32448	3.91284	3.57056	3.28203	3.03606	2.8243	2.64042	2.47959	2.33798
	J11	8.50674	7.0851	6.05931	5.28332	4.67579	4.18768	3.78738	3.45368	3.1717	2.93071	2.72277	2.54182	2.38324	2.24334
	J12	8.16956	6.80844	5.8262	5.08276	4.50011	4.03128	3.64605	3.32418	3.05153	2.81795	2.6159	2.43966	2.28485	2.148
	J13	7.77751	6.48535	5.55305	4.8474	4.29418	3.84869	3.48218	3.17545	2.9151	2.69155	2.49771	2.32822	2.17897	2.04672
	J14	7.32259	6.10892	5.23351	4.57108	4.05181	3.6336	3.28937	3.00103	2.75597	2.54518	2.36205	2.20155	2.05989	1.93404
	J15	6.79311	5.66945	4.85918	4.24623	3.76589	3.37911	3.06078	2.79409	2.5673	2.37205	2.20219	2.05307	1.9212	1.80378
	J16	6.17079	5.15178	4.41712	3.86152	3.42624	3.07588	2.78761	2.54615	2.34084	2.16404	2.01015	1.87494	1.75521	1.64844
	J17	5.4245	4.53001	3.88523	3.39767	3.01581	2.70852	2.45579	2.24418	2.0643	1.90946	1.77469	1.65626	1.55134	1.45771
	J18	4.4935	3.75347	3.22006	2.81678	2.50098	2.24691	2.038	1.86315	1.71457	1.58672	1.4755	1.37779	1.29124	1.21401
	J19	3.22212	2.69206	2.31003	2.02121	1.79508	1.61317	1.46364	1.33851	1.23221	1.14078	1.06127	0.991459	0.92965	0.874516
21	J1	17.1703	15.7801	14.8739	14.2532	13.8114	13.4869	13.2425	13.0544	12.9068	12.7892	12.6941	12.6163	12.5519	12.4982
	J2	12.0279	10.4384	9.3452	8.55929	7.97556	7.53081	7.18487	6.9111	6.69122	6.51236	6.36521	6.24293	6.14041	6.05378
	J3	11.0173	9.358	8.19348	7.33884	6.69079	6.18694	5.7873	5.46514	5.20185	4.98417	4.80239	4.64924	4.5192	4.40804
	J4	10.5986	8.91507	7.72284	6.83943	6.16281	5.63123	5.2051	4.85789	4.57109	4.33149	4.12934	3.95735	3.80992	3.68275
	J5	10.3502	8.6611	7.45881	6.5632	5.8734	5.32829	4.88866	4.52818	4.2285	3.97648	3.76245	3.57913	3.42096	3.28364
	J6	10.1614	8.47797	7.27541	6.37632	5.68129	5.13	4.68366	4.31623	4.00949	3.75044	3.52948	3.33938	3.17462	3.03092
	J7	9.98888	8.32041	7.12499	6.22856	5.53357	4.98074	4.5319	4.16135	3.85112	3.58835	3.36353	3.1695	3.0008	2.85319
	J8	9.81004	8.16542	6.98405	6.09578	5.40527	4.85462	4.40645	4.0356	3.7244	3.46021	3.23365	3.03766	2.86685	2.71703
	J9	9.61054	7.99848	6.8381	5.96349	5.28188	4.73697	4.29244	3.92376	3.61374	3.35001	3.1234	2.92699	2.75548	2.60475
	J10	9.38038	7.80923	6.67665	5.82132	5.15321	4.61781	4.17997	3.81602	3.50931	3.24786	3.02278	2.82734	2.65637	2.50586
	J11	9.11233	7.58992	6.49162	5.66106	5.01114	4.48919	4.06137	3.70492	3.40383	3.14662	2.92474	2.7317	2.56252	2.41333
	J12	8.80073	7.33459	6.27662	5.47602	4.84882	4.34431	3.92997	3.584	3.2911	3.04031	2.82348	2.63445	2.46845	2.32178
	J13	8.44043	7.03814	6.02635	5.26059	4.66038	4.17713	3.77972	3.4473	3.16531	2.92334	2.71367	2.53045	2.36921	2.22645
	J14	8.0254	6.69524	5.73572	5.00965	4.44054	3.98219	3.605	3.28916	3.02085	2.7902	2.58993	2.41456	2.25988	2.12262
	J15	7.54749	6.29904	5.3987	4.7176	4.18386	3.75404	3.40031	3.10398	2.85205	2.63525	2.44672	2.28134	2.13518	2.00520
	J16	6.99464	5.83957	5.00676	4.3769	3.88346	3.48623	3.15937	2.88559	2.65279	2.45236	2.27794	2.12476	1.98918	1.86841
	J17	6.34794	5.30117	4.54655	3.97595	3.52905	3.16938	2.87354	2.6258	2.4152	2.2339	2.0761	1.93747	1.81467	1.70518
	J18	5.57543	4.65718	3.9953	3.49488	3.10303	2.78774	2.52847	2.31144	2.12701	1.96829	1.83018	1.70885	1.60139	1.50554
	J19	4.61484	3.85562	3.30842	2.89475	2.57088	2.31031	2.0961	1.91684	1.76455	1.63353	1.51958	1.41952	1.33091	1.25190
	J20	3.30666	2.7632	2.37151	2.07545	1.84366	1.6572	1.50394	1.37571	1.2668	1.17313	1.09169	1.02021	0.956931	0.900535

Table III. Normalized admittance inverter values, $J_r = J_r/g$, for equal-resonator-Q_u absorptive bandstop filters and equiripple stopband attenuation L_S (J_0 is given by (10)).

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
2	qu	0.984726	0.708574	0.518897	0.383886	0.285667	0.213294	0.159557	0.119486	0.0895321	0.0671103	0.0503132	0.0377244	0.0282876	0.0212116
	3.01 dB	3.32156	4.51232	6.08486	8.16755	10.9329	14.6104	19.5069	26.0308	34.7261	46.3181	61.7738	82.3823	109.861	146.506
	2 dB	4.13403	5.65528	7.65649	10.3003	13.8053	18.4622	24.6597	32.9143	43.9147	58.5781	78.128	104.195	138.951	185.301
	1 dB	5.9215	8.16109	11.0948	14.9604	20.077	26.8692	35.9034	47.9327	63.9606	85.3237	113.804	151.778	202.408	269.928
	0.5 dB	8.42201	11.6542	15.8781	21.4359	28.7864	38.5393	51.5079	68.7735	91.7763	122.435	163.306	217.799	290.456	387.348
3	qu	1.89529	1.44730	1.13210	0.900258	0.724114	0.587102	0.478657	0.391752	0.321481	0.264295	0.217560	0.179243	0.147761	0.121857
	3.01 dB	2.32616	2.91915	3.63548	4.49696	5.53192	6.77583	8.27303	10.0775	12.2552	14.8863	18.0671	21.9155	26.5734	32.2129
	2 dB	2.81099	3.57034	4.48145	5.57178	6.87715	8.4423	10.323	12.5871	15.3174	18.6143	22.5988	27.4181	33.2502	40.3106
	1 dB	3.90135	5.02447	6.36114	7.95196	9.84942	12.1188	14.841	18.1143	22.0584	26.8186	32.5693	39.5232	47.937	58.1216
	0.5 dB	5.45575	7.08219	9.00856	11.2941	14.0148	17.2644	21.159	25.8392	31.4761	38.2776	46.4928	56.4255	68.4425	82.9876
4	qu	2.84833	2.23007	1.79171	1.46629	1.21650	1.01972	0.861435	0.732049	0.624943	0.535355	0.459830	0.395744	0.341115	0.294361
	3.01 dB	1.93142	2.32780	2.79069	3.32621	3.94202	4.64799	5.45682	6.38353	7.4458	8.66487	10.0651	11.6754	13.5283	15.6626
	2 dB	2.28375	2.79456	3.3859	4.06549	4.84313	5.73142	6.74643	7.90711	9.23562	10.7586	12.5065	14.5153	16.8259	19.4863
	1 dB	3.09075	3.85323	4.72606	5.72128	6.85375	8.14228	9.61049	11.2859	13.2008	15.3934	17.9078	20.7959	24.1161	27.9378
	0.5 dB	4.26008	5.37103	6.63382	8.06702	9.69281	11.5387	13.6388	16.0327	18.7665	21.8952	25.4814	29.5993	34.3323	39.7791
5	qu	3.82375	3.03559	2.47456	2.05627	1.73353	1.47792	1.27116	1.10101	0.959002	0.839083	0.736851	0.648979	0.572936	0.506761
	3.01 dB	1.71881	2.01803	2.362	2.75253	3.19224	3.68455	4.23398	4.84618	5.52788	6.28702	7.13256	8.07504	9.12645	10.3003
	2 dB	1.99803	2.3863	2.82819	3.32592	3.88294	4.50372	5.19412	5.96138	6.81400	7.76198	8.81655	9.99089	11.3000	12.7607
	1 dB	2.64753	3.23301	3.89025	4.62324	5.43768	6.34063	7.34101	8.44957	9.67882	11.0433	12.5592	14.2457	16.1242	18.2190
	0.5 dB	3.60229	4.46128	5.41683	6.47603	7.64807	8.94368	10.3761	11.9610	13.7163	15.6632	17.8247	20.2281	22.9041	25.8873
6	qu	4.81317	3.85527	3.17162	2.66061	2.26532	1.95121	1.69626	1.48575	1.30940	1.15986	1.03176	0.921034	0.824602	0.740075
	3.01 dB	1.58558	1.82666	2.1016	2.4106	2.75429	3.13405	3.55171	4.00961	4.51072	5.05861	5.65729	6.3116	7.02692	7.80928
	2 dB	1.81789	2.13281	2.48805	2.88376	3.32084	3.80116	4.32722	4.90207	5.52957	6.21425	6.96122	7.77656	8.66699	9.64006
	1 dB	2.36548	2.84515	3.37782	3.96427	4.60647	5.30774	6.07212	6.90439	7.81039	8.79682	9.87115	11.0422	12.3198	13.7148
	0.5 dB	3.18089	3.88967	4.66822	5.51902	6.44592	7.45436	8.55063	9.74191	11.0368	12.445	13.9773	15.6465	17.4664	19.4526
7	qu	5.81230	4.68447	3.87850	3.27483	2.80692	2.43447	2.13154	1.88084	1.67035	1.49136	1.33764	1.20439	1.08803	0.985618
	3.01 dB	1.49412	1.69643	1.92604	2.18264	2.46608	2.77661	3.115	3.48228	3.87982	4.30958	4.77339	5.27376	5.81317	6.39523
	2 dB	1.69354	1.95943	2.25776	2.58797	2.94991	3.34403	3.77142	4.23355	4.73224	5.27004	5.84932	6.47329	7.14506	7.86919
	1 dB	2.16903	2.57793	3.02886	3.52146	4.05608	4.63393	5.25706	5.92791	6.64943	7.42547	8.25963	9.15663	10.121	11.1594
	0.5 dB	2.88537	3.49386	4.15653	4.87417	5.64831	6.48137	7.3768	8.33849	9.37091	10.4798	11.6703	12.9495	14.3237	15.8026
8	qu	6.81816	5.52077	4.59231	3.89604	3.35570	2.92495	2.57406	2.28325	2.03867	1.83042	1.65123	1.49559	1.35938	1.23931
	3.01 dB	1.42745	1.6019	1.79944	2.01943	2.26132	2.52495	2.81052	3.11837	3.44919	3.80388	4.18352	4.58956	5.02333	5.48666
	2 dB	1.60242	1.83298	2.091	2.37544	2.6856	3.02138	3.38315	3.77148	4.18734	4.63196	5.1068	5.61369	6.15438	6.73117
	1 dB	2.02385	2.38163	2.77468	3.20178	3.66242	4.15694	4.68633	5.25175	5.8549	6.49777	7.18262	7.91223	8.68923	9.51699
	0.5 dB	2.6655	3.20158	3.78233	4.40723	5.07651	5.79136	6.55374	7.36569	8.22995	9.14954	10.1279	11.169	12.2768	13.4562
9	qu	7.82927	6.36200	5.31132	4.52265	3.90997	3.42089	3.02213	2.69124	2.41257	2.17515	1.97051	1.79255	1.63657	1.49891
	3.01 dB	1.37664	1.53015	1.7037	1.89655	2.10798	2.33768	2.58541	2.85126	3.13555	3.43848	3.76094	4.10362	4.46735	4.85309
	2 dB	1.53267	1.73657	1.96437	2.21485	2.48706	2.78066	3.09548	3.43171	3.7899	4.17037	4.57433	5.00271	5.45661	5.93725
	1 dB	1.91183	2.23092	2.58052	2.95904	3.36553	3.79991	4.26236	4.75351	5.2744	5.82574	6.40946	7.02702	7.68011	8.3706
	0.5 dB	2.49474	2.97604	3.49535	4.05156	4.6442	5.27389	5.94138	6.648	7.39553	8.18519	9.01991	9.90192	10.8337	11.8181
10	qu	8.84453	7.20765	6.03450	5.15344	4.46830	3.92100	3.47449	3.10354	2.79091	2.52422	2.29419	2.09394	1.91827	1.76304
	3.01 dB	1.33663	1.47374	1.62867	1.80057	1.98874	2.19264	2.41188	2.64643	2.89627	3.16151	3.44257	3.73987	4.05386	4.3852
	2 dB	1.47752	1.66046	1.86474	2.08897	2.33218	2.59373	2.8732	3.17066	3.48619	3.82001	4.17272	4.54495	4.93728	5.35059
	1 dB	1.8226	2.11115	2.42688	2.76785	3.13297	3.52173	3.93385	4.3698	4.82996	5.31486	5.82555	6.36308	6.92841	7.52289
	0.5 dB	2.35785	2.79586	3.26733	3.7705	4.30469	4.86985	5.46611	6.09455	6.75599	7.45144	8.18256	8.951	9.75823	10.6063

Table IV. Reflection-mode-prototype resonator q and 3.01 dB, 2 dB, 1 dB, and 0.5 dB passband-edge frequencies for equiripple reflection-stopband attenuation L_s=L_h at normalized stopband edge frequencies ω_h=±1.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
11	qu	9.86305	8.05652	6.76115	5.78756	5.03004	4.42461	3.93022	3.51929	3.17269	2.87675	2.62137	2.39891	2.20355	2.03077
	3.01 dB	1.30431	1.42824	1.56822	1.72346	1.89318	2.07677	2.27381	2.48406	2.70743	2.9439	3.19358	3.45674	3.73368	4.02481
	2 dB	1.43279	1.59883	1.78418	1.9875	2.20769	2.44399	2.69593	2.96331	3.24609	3.54433	3.85825	4.18824	4.53478	4.89837
	1 dB	1.74974	2.01354	2.30195	2.61297	2.94527	3.2981	3.67109	4.06429	4.47788	4.91217	5.36767	5.84509	6.34522	6.86892
	0.5 dB	2.2454	2.64829	3.08116	3.54207	4.02995	4.54435	5.0853	5.65324	6.24875	6.87251	7.52541	8.20862	8.92338	9.67099
12	qu	10.8844	8.90817	7.49048	6.42448	5.59459	4.93107	4.38871	3.93779	3.55729	3.23220	2.95147	2.70677	2.49171	2.30147
	3.01 dB	1.27764	1.39073	1.51848	1.66009	1.81481	1.98196	2.16118	2.35204	2.55435	2.76805	2.99313	3.22971	3.47802	3.73479
	2 dB	1.39577	1.54785	1.71766	1.90383	2.10528	2.32115	2.55099	2.79436	3.0511	3.32119	3.60472	3.9019	4.21306	4.53554
	1 dB	1.68904	1.93232	2.19822	2.48467	2.79025	3.114	3.45561	3.81471	4.19133	4.58564	4.99795	5.42873	5.87856	6.34571
	0.5 dB	2.15117	2.5249	2.92596	3.35222	3.80241	4.27579	4.77244	5.29218	5.8354	6.40257	6.99432	7.61145	8.25492	8.9238
13	qu	11.9080	9.76204	8.22212	7.06371	6.16150	5.43966	4.84964	4.35874	3.94430	3.59001	3.28394	3.01704	2.78237	2.57460
	3.01 dB	1.25527	1.35928	1.4768	1.60706	1.74932	1.90294	2.06741	2.24237	2.42755	2.62282	2.82807	3.04334	3.26872	3.50432
	2 dB	1.36461	1.50498	1.66174	1.8336	2.01947	2.21849	2.43004	2.65373	2.88929	3.13662	3.39567	3.66653	3.94939	4.24442
	1 dB	1.63766	1.86361	2.11058	2.37647	2.6598	2.95961	3.27524	3.60642	3.953	4.31502	4.6926	5.086	5.49564	5.92185
	0.5 dB	2.07097	2.42002	2.79431	3.19158	3.61043	4.05006	4.51006	4.99039	5.49116	6.01268	6.55527	7.1195	7.70605	8.31549
14	qu	12.9335	10.6179	8.95572	7.70485	6.73036	5.95035	5.31252	4.78167	4.33329	3.94988	3.61845	3.32931	3.07506	2.84979
	3.01 dB	1.23623	1.33253	1.44136	1.56202	1.69375	1.83596	1.98809	2.14977	2.32071	2.50068	2.6896	2.88741	3.09406	3.30971
	2 dB	1.33802	1.46839	1.61405	1.77377	1.94645	2.13124	2.32748	2.53473	2.75269	2.98114	3.22004	3.46936	3.72912	3.99955
	1 dB	1.59357	1.80466	2.03546	2.28387	2.54836	2.82793	3.12185	3.42972	3.75134	4.0866	4.43561	4.79847	5.17533	5.56661
	0.5 dB	2.00178	2.32962	2.68103	3.05367	3.44598	3.8571	4.28649	4.73395	5.19949	5.6832	6.1854	6.70643	7.24658	7.80658
15	qu	13.9607	11.4755	9.69102	8.34776	7.30086	6.46276	5.77712	5.20631	4.72403	4.31146	3.95471	3.64338	3.36945	3.12673
	3.01 dB	1.21983	1.30949	1.41086	1.52326	1.64599	1.77843	1.92006	2.07047	2.22934	2.39645	2.57164	2.75482	2.94595	3.14503
	2 dB	1.31506	1.43678	1.57288	1.72214	1.88351	2.05612	2.23932	2.43261	2.63566	2.84823	3.0702	3.30149	3.54211	3.79211
	1 dB	1.55531	1.75349	1.97029	2.20362	2.45194	2.71416	2.98958	3.27768	3.57819	3.89096	4.21598	4.55328	4.90302	5.26532
	0.5 dB	1.94141	2.25079	2.58237	2.93374	3.3033	3.69001	4.09335	4.51294	4.9487	5.40066	5.86898	6.35387	6.85568	7.37468
16	qu	14.9895	12.3326	10.4264	8.99088	7.87289	6.97576	6.24323	5.63247	5.11628	4.67456	4.29250	3.95902	3.66542	3.40527
	3.01 dB	1.20555	1.2895	1.38438	1.48964	1.60447	1.72859	1.86103	2.00174	2.15027	2.30637	2.46986	2.64058	2.81857	3.00364
	2 dB	1.29503	1.4093	1.53706	1.67725	1.82866	1.99089	2.16265	2.34393	2.53419	2.73318	2.94071	3.15665	3.38108	3.61382
	1 dB	1.52175	1.70878	1.91333	2.13354	2.36758	2.61505	2.8742	3.14527	3.42769	3.72123	4.02583	4.34141	4.6682	5.00606
	0.5 dB	1.88823	2.18163	2.49585	2.82871	3.17813	3.54412	3.92453	4.32015	4.73041	5.15525	5.59475	6.04895	6.51832	7.00276
17	qu	16.0194	13.1948	11.1658	9.63774	8.44623	7.49181	6.71063	6.05992	5.50981	5.03896	4.63159	4.27587	3.96271	3.68506
	3.01 dB	1.19301	1.27183	1.36101	1.45995	1.56803	1.68465	1.8093	1.94157	2.08112	2.22769	2.38108	2.54114	2.70778	2.88092
	2 dB	1.2774	1.38494	1.50534	1.6375	1.78041	1.93323	2.09532	2.26613	2.44529	2.63251	2.82759	3.03039	3.24085	3.45891
	1 dB	1.4921	1.66896	1.86267	2.07123	2.29309	2.52714	2.77257	3.02882	3.29553	3.57244	3.85942	4.15641	4.46343	4.7805
	0.5 dB	1.84102	2.11976	2.41857	2.73499	3.0673	3.41439	3.77555	4.15031	4.53845	4.93984	5.35449	5.78246	6.2239	6.67896
18	qu	17.0505	14.0563	11.9049	10.2845	9.02068	8.00810	7.17917	6.48851	5.904520	5.40453	4.97186	4.59395	4.26119	3.966050
	3.01 dB	1.18191	1.25623	1.34038	1.43376	1.53579	1.6459	1.76358	1.88842	2.0201	2.15832	2.30289	2.45364	2.61045	2.77326
	2 dB	1.26176	1.3634	1.47729	1.60234	1.73761	1.88228	2.03568	2.19728	2.3667	2.54362	2.72782	2.91916	3.11751	3.32284
	1 dB	1.46569	1.6336	1.81767	2.0159	2.22679	2.44919	2.68229	2.9255	3.17842	3.44077	3.71238	3.99315	4.28306	4.58213
	0.5 dB	1.79877	2.0646	2.34971	2.65154	2.9684	3.29912	3.64294	3.99936	4.36809	4.74898	5.14196	5.54706	5.96435	6.39398

Table IV. Reflection-mode-prototype resonator q and 3.01 dB, 2 dB, 1 dB, and 0.5 dB passband-edge frequencies for equiripple reflection-stopband attenuation L_s=L_h at normalized stopband edge frequencies ω_h=±1.

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB	50 dB	55 dB	60 dB	65 dB	70 dB	75 dB	80 dB	85 dB
19	qu	18.0825	14.9186	12.6451	10.9323	9.59615	8.52543	7.64875	6.91812	6.30025	5.77112	5.31317	4.91307	4.56070	4.24811
	3.01 dB	1.17202	1.24234	1.32198	1.41041	1.50706	1.61137	1.72286	1.84113	1.96582	2.09668	2.23347	2.37603	2.52423	2.67797
	2 dB	1.24779	1.34417	1.45221	1.57093	1.69938	1.83677	1.98245	2.13589	2.29667	2.4645	2.63911	2.82036	3.00811	3.20228
	1 dB	1.44201	1.6019	1.77729	1.96628	2.16734	2.37935	2.60148	2.83312	3.07382	3.32332	3.58138	3.84792	4.12283	4.40613
	0.5 dB	1.76073	2.01496	2.2877	2.57645	2.87949	3.19561	3.524	3.86417	4.21573	4.57853	4.95242	5.33744	5.73358	6.14095
20	qu	19.1156	15.7820	13.3862	11.5810	10.1725	9.04367	8.11921	7.34868	6.69689	6.13865	5.65540	5.23312	4.86115	4.53111
	3.01 dB	1.16314	1.22987	1.30548	1.38946	1.48128	1.5804	1.68635	1.79874	1.91722	2.04151	2.17139	2.30669	2.44726	2.59299
	2 dB	1.23524	1.32687	1.42967	1.54268	1.66501	1.79588	1.93463	2.08076	2.23385	2.39357	2.55966	2.73197	2.91032	3.09464
	1 dB	1.42064	1.57328	1.74085	1.92149	2.11372	2.31639	2.52868	2.74995	2.97977	3.21782	3.46385	3.71775	3.9794	4.24878
	0.5 dB	1.72628	1.96997	2.23154	2.50849	2.79909	3.10208	3.41666	3.74225	4.07852	4.4252	4.78216	5.14939	5.52683	5.91456
21	qu	20.1494	16.6461	14.1280	12.2304	10.7497	9.56273	8.59052	7.78006	7.09438	6.50702	5.99846	5.55401	5.16243	4.81502
	3.01 dB	1.15513	1.21862	1.29059	1.37056	1.45802	1.55246	1.65343	1.76052	1.87341	1.99183	2.11553	2.24434	2.3781	2.51667
	2 dB	1.2239	1.31124	1.40929	1.51715	1.63394	1.75891	1.89143	2.03097	2.17713	2.32958	2.48805	2.65236	2.82234	2.99786
	1 dB	1.40128	1.54732	1.70779	1.88086	2.06507	2.2593	2.4627	2.67464	2.89467	3.12244	3.35772	3.60033	3.85017	4.10711
	0.5 dB	1.69494	1.92902	2.18042	2.44666	2.72595	3.01708	3.31918	3.63167	3.95418	4.28642	4.62825	4.97958	5.34038	5.71056
22	qu	21.1796	17.5110	14.8707	12.8807	11.3276	10.0826	9.06258	8.21217	7.49262	6.87614	6.34228	5.87567	5.46449	5.09956
	3.01 dB	1.14791	1.20842	1.27708	1.35342	1.43693	1.52712	2.93942	1.72588	1.83373	1.94683	2.06496	2.18792	2.31557	2.44778
	2 dB	1.21367	1.29704	1.39077	1.49394	1.6057	1.72531	1.85217	1.98576	2.12566	2.27154	2.42314	2.58025	2.74271	2.9104
	1 dB	1.38375	1.52365	1.67763	1.8438	2.02072	2.20725	2.4026	2.60609	2.81726	3.03576	3.26133	3.49379	3.73302	3.97892
	0.5 dB	1.66646	1.89155	2.13365	2.39009	2.65911	2.93942	3.23021	3.53085	3.84091	4.16012	4.48831	4.82535	5.1712	5.52584
23	qu	22.2192	18.3765	15.6140	13.5316	11.9063	10.6030	9.53531	8.64496	7.89152	7.24592	6.68677	6.19800	5.76722	5.38485
	3.01 dB	1.14126	1.19913	1.26478	1.3378	1.41771	1.50404	1.59638	1.68227	1.79759	1.90588	2.01896	2.13664	2.25877	2.3852
	2 dB	1.20422	1.28409	1.37387	1.47275	1.57991	1.69465	1.81635	1.93408	2.07872	2.21864	2.36401	2.51461	2.67027	2.83087
	1 dB	1.3675	1.50199	1.65002	1.80986	1.98009	2.15963	2.3476	2.53558	2.7465	2.95659	3.17337	3.39664	3.62628	3.86218
	0.5 dB	1.63996	1.85714	2.09068	2.33814	2.59772	2.86819	3.14864	3.4328	3.73722	4.04462	4.36045	4.68456	5.01692	5.35747
24	qu	23.2550	19.2427	16.3579	14.1831	12.4855	11.1242	10.0087	9.07840	8.29107	7.61635	7.03192	6.52096	6.07059	5.67077
	3.01 dB	1.13521	1.19063	1.25353	1.32351	1.40012	1.48291	1.57149	1.66547	1.76454	1.86844	1.97691	2.08979	2.20689	2.32657
	2 dB	1.19562	1.27222	1.35838	1.45333	1.55628	1.66653	1.78351	1.90671	2.03571	2.1702	2.30988	2.45456	2.60404	2.75687
	1 dB	1.35269	1.48208	1.62463	1.77865	1.94274	2.11581	2.29704	2.48578	2.68153	2.88393	3.09269	3.30762	3.52855	3.75436
	0.5 dB	1.61571	1.82541	2.05106	2.29024	2.54115	2.80253	3.07352	3.35345	3.64188	3.93848	4.24305	4.55543	4.87554	5.20257
25	qu	24.2914	20.1094	17.1024	14.8352	13.0653	11.6458	10.4826	9.51239	8.69118	7.98734	7.37762	6.84450	6.37454	5.95736
	3.01 dB	1.12966	1.18283	1.24319	1.31038	1.38396	1.46352	1.54863	1.63896	1.7342	1.83406	1.93833	2.04681	2.15933	2.27574
	2 dB	1.18772	1.26131	1.34414	1.43546	1.53453	1.64067	1.75329	1.87193	1.99616	2.12566	2.26014	2.3994	2.54324	2.69151
	1 dB	1.33903	1.46372	1.6012	1.74984	1.90826	2.0754	2.25041	2.43266	2.62164	2.817	3.01842	3.2257	3.43869	3.6572
	0.5 dB	1.59328	1.79605	2.01439	2.2459	2.4888	2.74184	3.00409	3.27492	3.55386	3.84058	4.13485	4.43649	4.74542	5.06149
26	qu	25.3283	20.9767	17.8474	15.4878	13.6456	12.168	10.9570	9.94691	9.09182	8.35887	7.72386	7.16857	6.67902	6.24431
	3.01 dB	1.12455	1.17563	1.23366	1.29828	1.36907	1.44563	1.52756	1.61453	1.70622	1.80239	1.90278	2.00723	2.11555	2.22761
	2 dB	1.18043	1.25124	1.33099	1.41896	1.51444	1.61677	1.7254	1.83982	1.95965	2.08455	2.21425	2.34853	2.4872	2.63012
	1 dB	1.32639	1.44673	1.57951	1.72316	1.87633	2.03797	2.20724	2.38349	2.56624	2.7551	2.94978	3.15005	3.35574	3.56672
	0.5 dB	1.57248	1.76879	1.98034	2.20474	2.44022	2.6855	2.9397	3.20212	3.47231	3.74993	4.03472	4.32652	4.62519	4.93068

Table IV. Reflection-mode-prototype resonator q and 3.01 dB, 2 dB, 1 dB, and 0.5 dB passband-edge frequencies for equiripple reflection-stopband attenuation L_s=L_h at normalized stopband edge frequencies ω_h=±1.

Synthesis of Lossy Reflection-Mode Bandstop Filters

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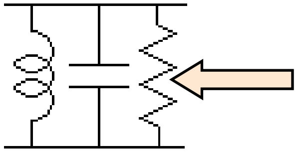
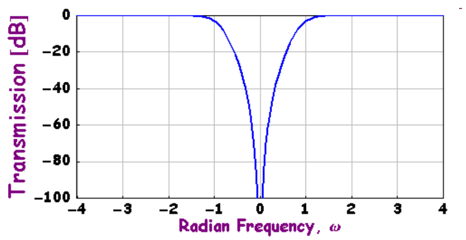
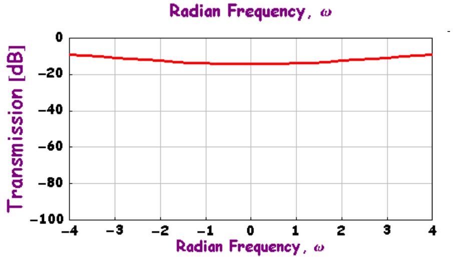
CNES, Toulouse, France, 16-18 October 2006

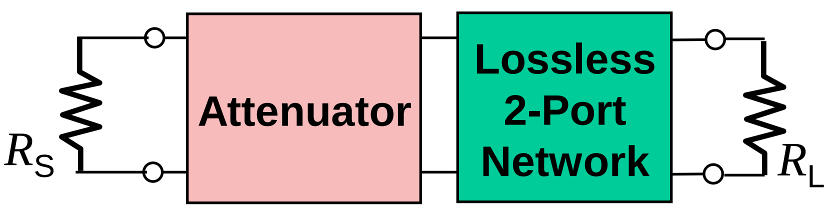
16 October 2006

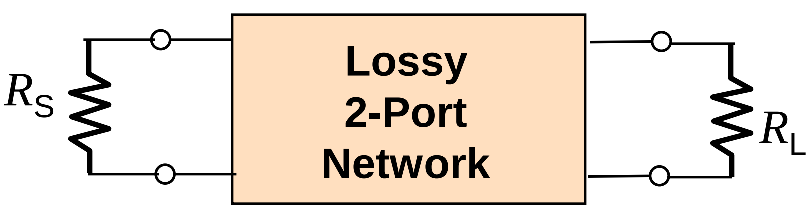
Lossy Filters











- Real filter components are lossy
- Conventional filter synthesis assumes lossless components
- Synthesis works out very nicely by ignoring loss
- Filter performance does not work out very nicely by ignoring loss
- Predistortion techniques evolved to “correct” for loss – but increase passband loss level
- Lossy reflection-mode synthesis corrects stopband attenuation without increasing passband loss

Lossy Bandstop Filters

Lossless [Blue]
vs:

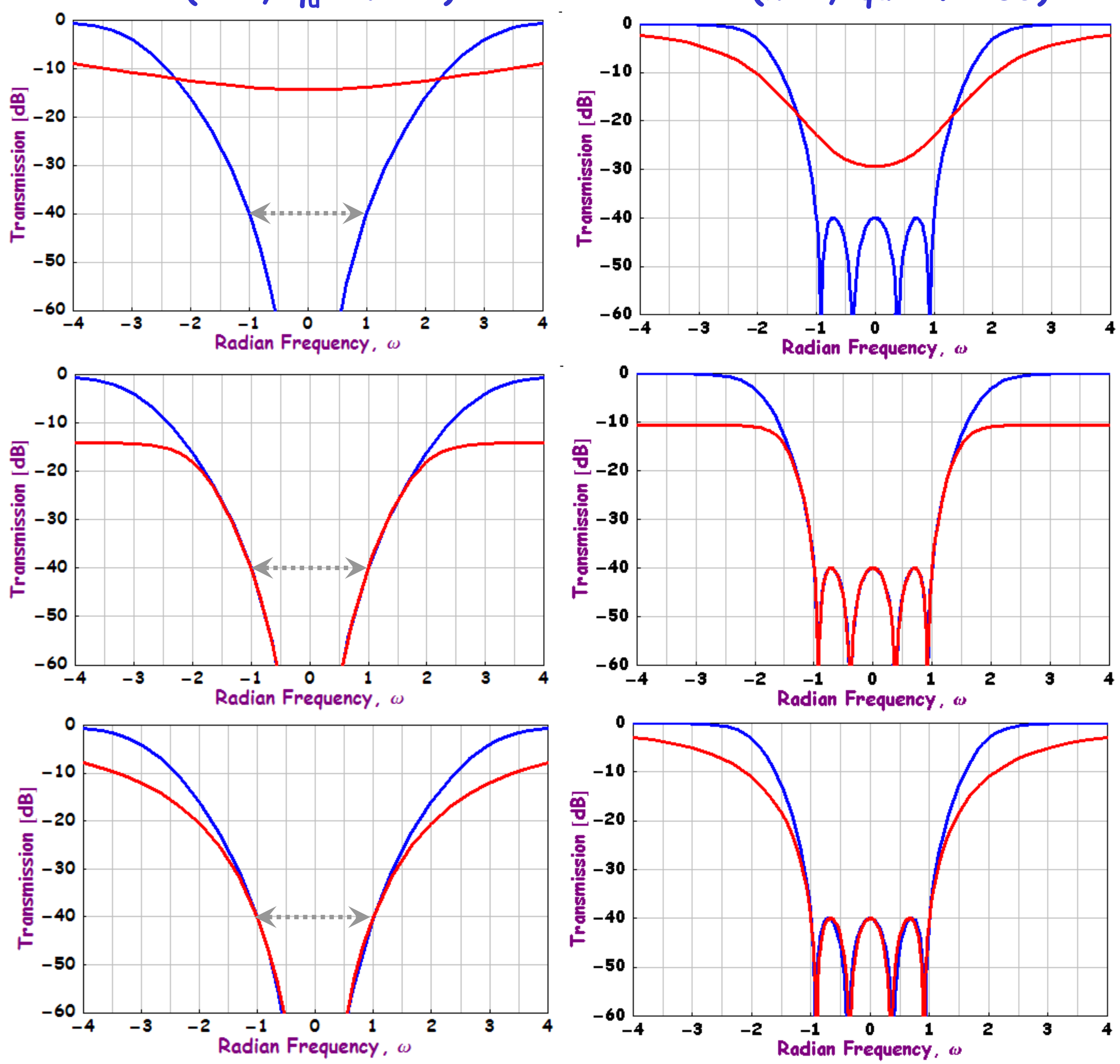
Maximally Flat (n=4, q_u=0.666)

Equiripple (n=4, qu=1.2165)

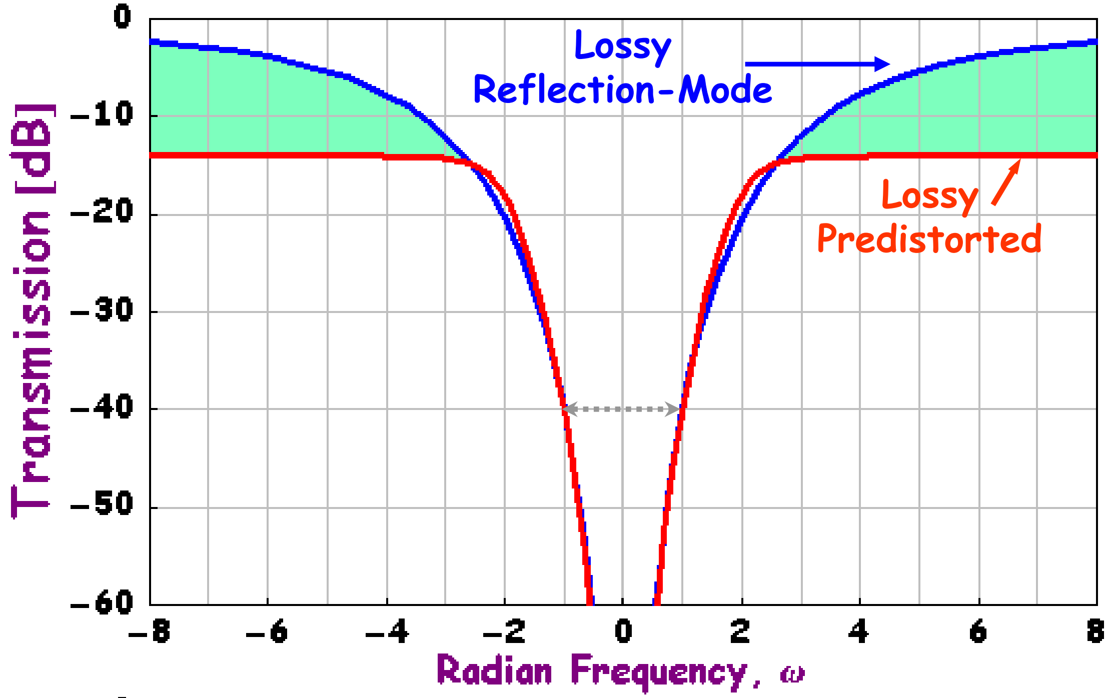
Lossy Conventional [Red]

Lossy Predistorted [Red]

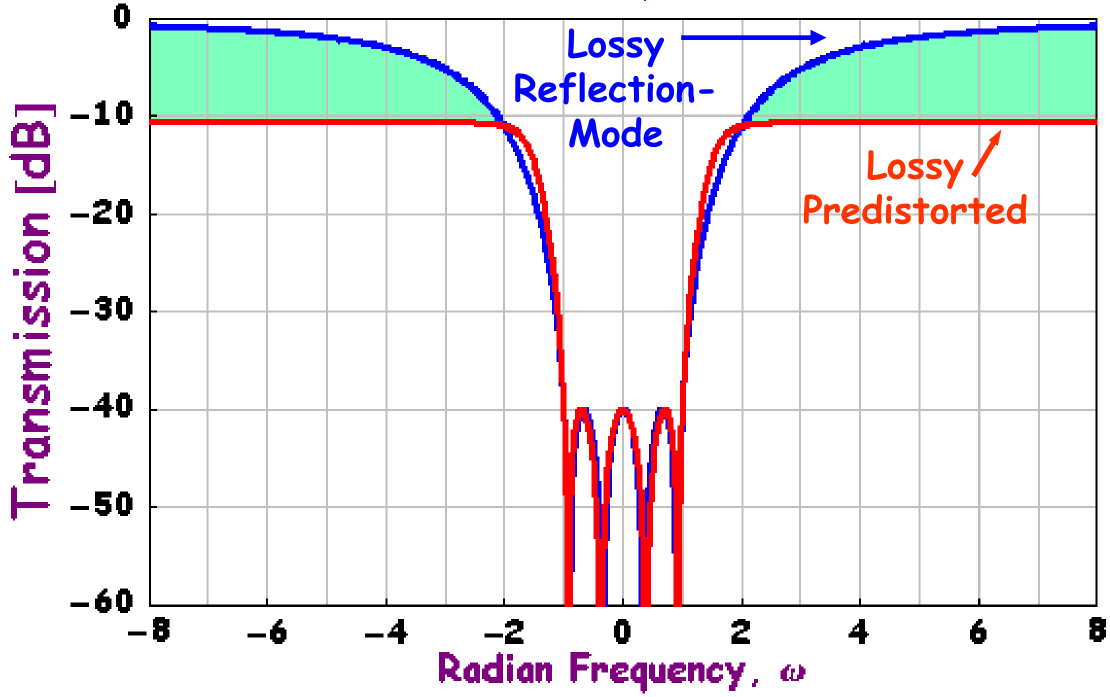
Lossy Absorptive (Reflection-Mode) [Red]



Lossy Reflection-Mode vs. Predistorted



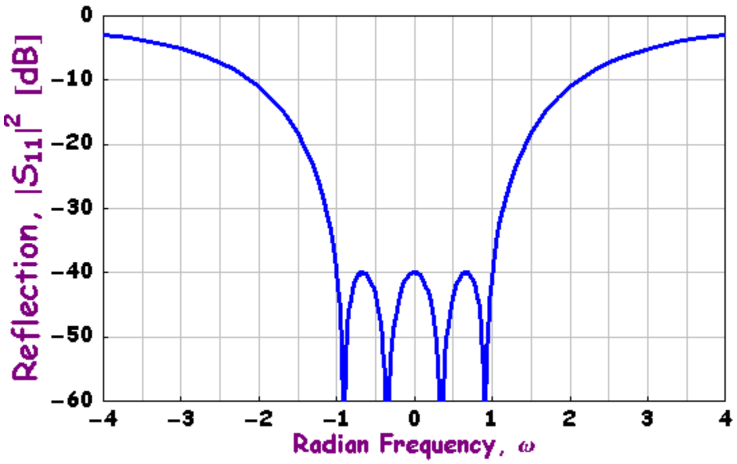
Maximally Flat
($n=4$, $q_u=0.666$)



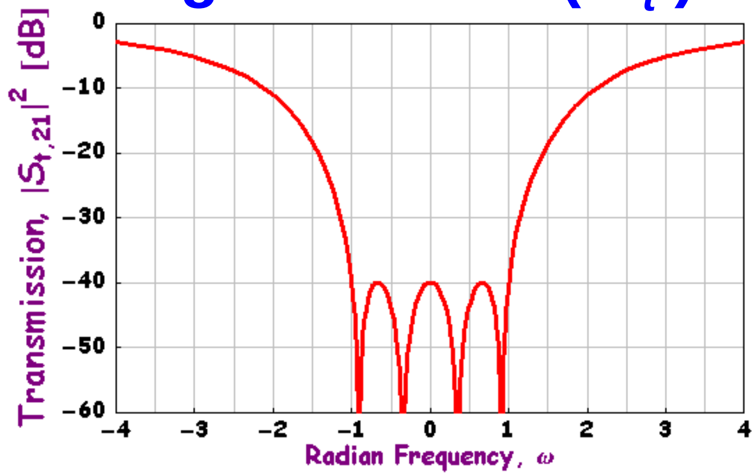
Equiripple
($n=4$, $qu=1.2165$)

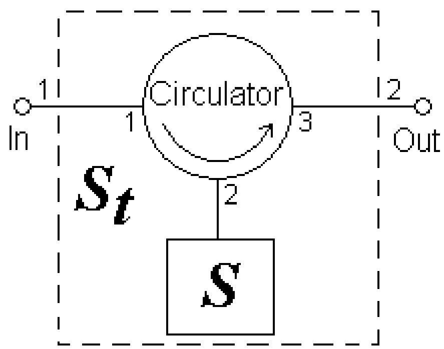
Reflection-Mode Filters

- **Convert reflection characteristics of a sub-network (*S*) into transmission characteristics of a larger network (S_t)**

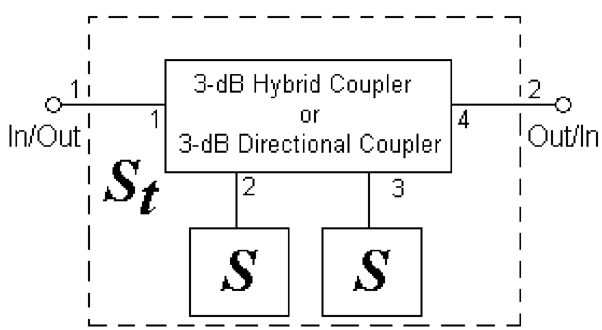


S_{11} **to** $S_{t,21}$





- **Non-reciprocal circulator based**

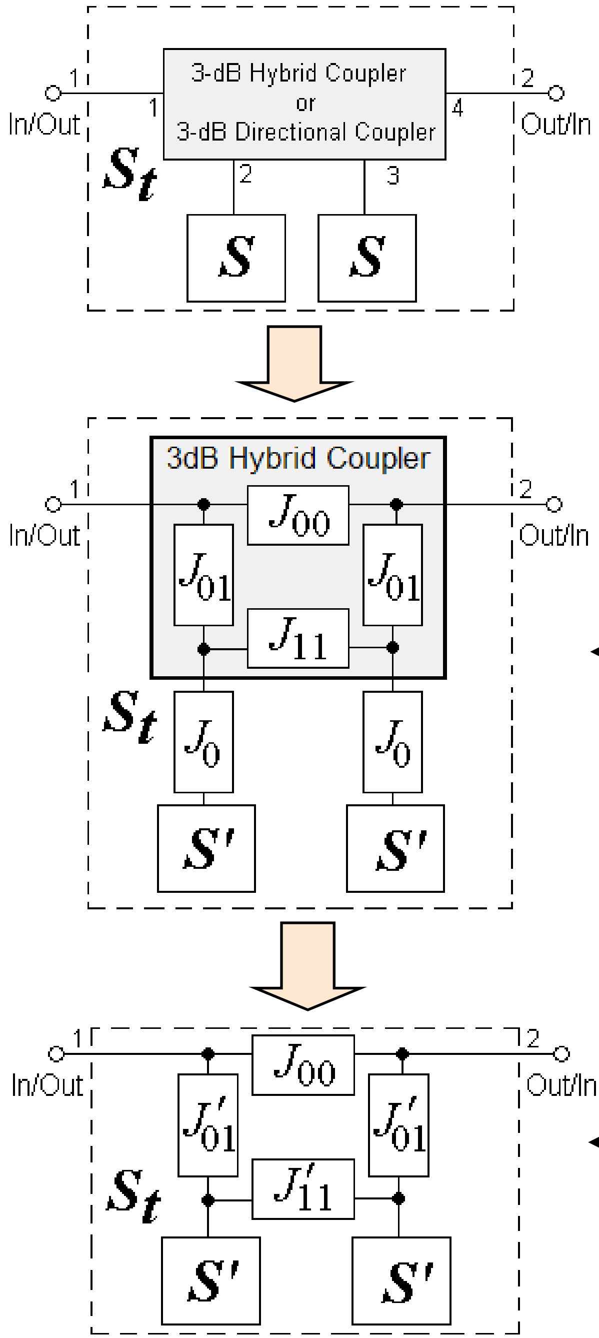


- **Reciprocal 3-dB hybrid coupler or a 3-dB directional coupler based**

Alternative Reflection-Mode Network Topologies

- **Other topologies can be derived using circuit transformations**

- **Example: Portions of one-port networks S can be absorbed into a hybrid coupler [10], resulting in truncated one-port networks S'**



If:

$$Y_s = \text{Source \& Load Admittance}$$

$$J_{00} = J_{11} = Y_s$$

$$J_{01} = \sqrt{2}\ J_{00} = \sqrt{2}\ Y_s$$

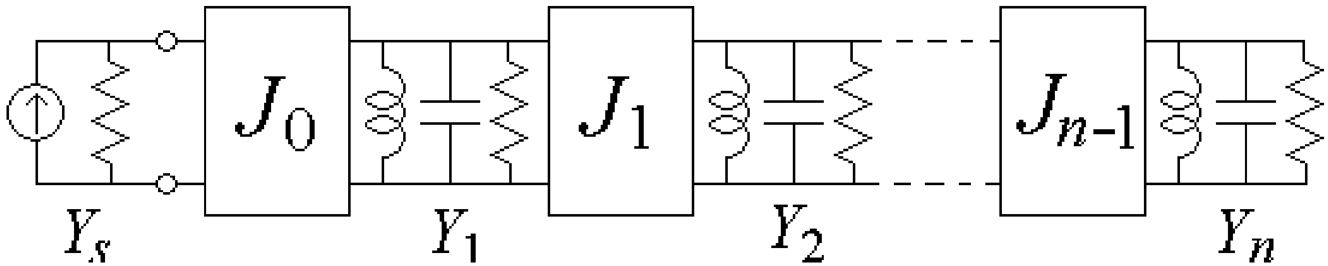
Then:

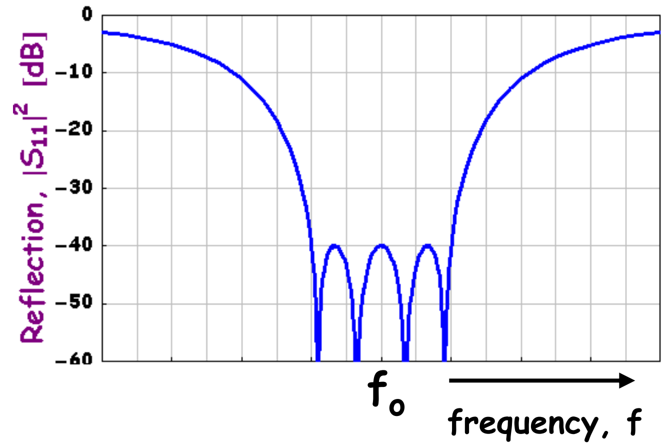
$$J'_{01} = \sqrt{2}\ J_0$$

$$J'_{11} = J_0^2 \big/ Y_s$$

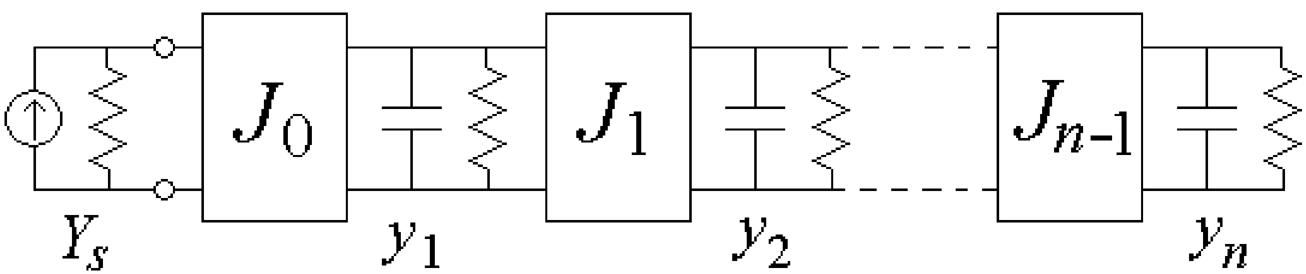
Lossy Reflection-Mode Ladder Networks

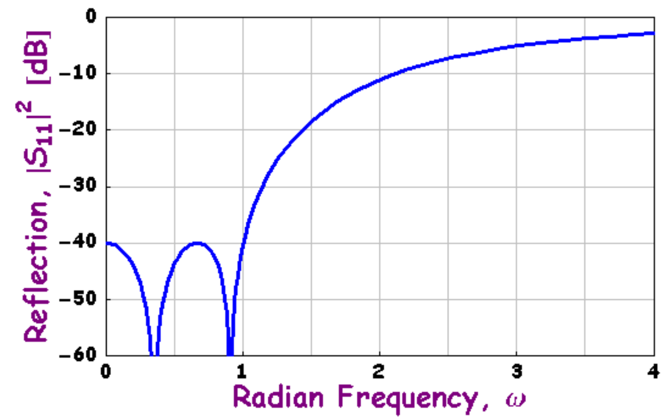
- S could be any network
- For simplicity, let it be a one-port ladder network





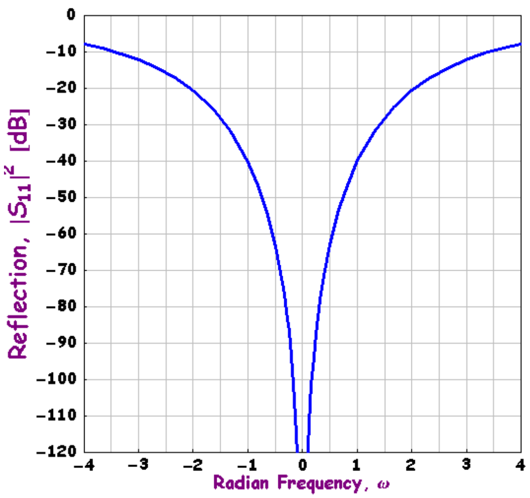
One-Port "Bandstop" Reflection Network





Corresponding One-Port "Highpass" Reflection Prototype

Generic Maximally-Flat Reflection Prototype



$$S_{11}(s) = \pm s^n \prod_{r=1}^{n} \left(s - j\sigma_o \, e^{j\theta_r} \right)^{-1}$$

with poles on circle of radius σ_o centered at origin in complex s plane (i.e., σ_o scales filter bandwidth):

$$s_r = j\sigma_o \, e^{j\theta_r} = \sigma_o \left(-\sin\theta_r + j\cos\theta_r \right) \quad \text{for} \quad r = 1 \ldots n$$

where: $$\theta_r = \theta \, \frac{(2r + x - (n+1))}{2}, \quad \theta = \frac{\pi}{x}, \quad \text{and } x \geq n$$

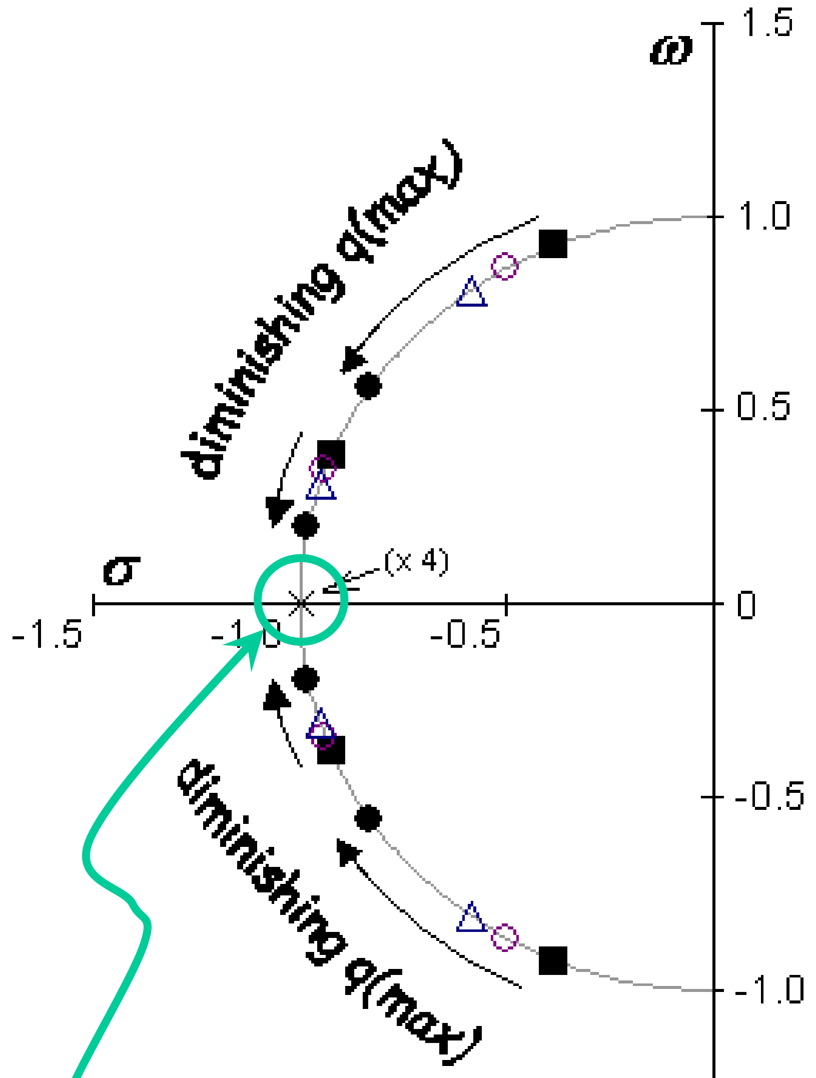
(θ is the constant angular separation between the poles)

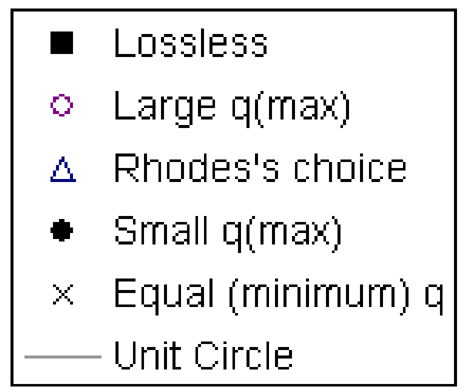
x	θ_r	θ	Description
∞	$\pi / 2$	0	equal resonator Q_u
$2n$	$\pi(2r+(n-1))/(4n)$	$\pi / (2n)$	small range in Q_u
$n+1$	$\pi r / (n+1)$	$\pi / (n+1)$	Rhodes' choice [5]
$n+½$	$\pi(4r-1) / (2(2n+1))$	$2\pi/(2n+1)$	large range in Q_u
n	$\pi(2r-1)/(2n)$	π / n	infinite Q_u

Between extremes x=n and x=infinity ($\theta=\pi/$n and $\theta=0$) is the continuum of possible maximally-flat characteristics for unequal-Q_u reflection filters

Generic Maximally-Flat Reflection Prototype

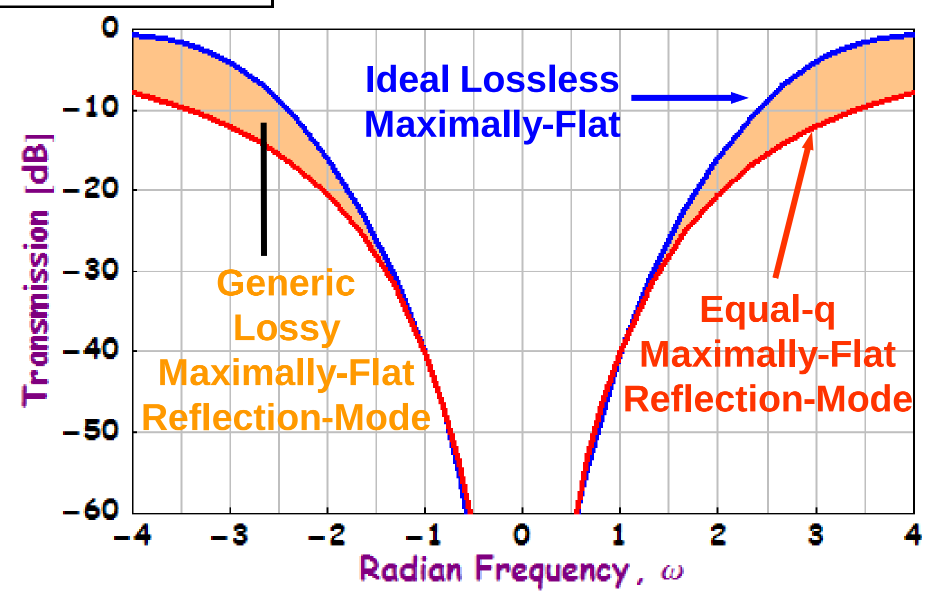
Left-Half-Plane Reflection Poles for n = 4





Reflection poles for 5 different choices of x (5 different sets of shunt admittance q), showing the trend in the 4 pole locations as the maximum q diminishes

Poles collapse to a single point (σ_o=-1) for the equal-q (minimum q_{max}) case



Synthesis of Generic Maximally-Flat Prototype

$$S_{11}(s) = \pm s^n \left(s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0\right)^{-1}$$

Input admittance, Y_{in}, is given by:

$$Y_{in} = \frac{1-S_{11}}{1+S_{11}} = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0}{2s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0}$$

Using Linnebach's technique [12], it is possible to derive:

$$a_0 = \sigma_o^n \text{ and } a_r = \frac{a_{n-r}}{\sigma_o^{2r-n}} = \sigma_o^{n-r} \prod_{\mu=1}^{r} \frac{\cos\left(\frac{\pi}{2x}\left(x+\mu-(n+1)\right)\right)}{\sin\left(\frac{\pi}{2x}\mu\right)}$$

Synthesis of generic maximally-flat reflection prototypes is accomplished by sequentially extracting element values (ideal admittance inverters and lossy shunt admittances) from Y_{in} using continued fraction expansion [11].

Synthesis of Generic Maximally-Flat Prototype

Maximum unloaded *Q* of maximally-flat prototype's shunt admittances is:

$$q_{\max} = q_1 = \frac{\omega_h c_1}{g_1} = \frac{2\omega_h}{a_{n-1} - 2\left(a_{n-2}/a_{n-1}\right)}$$

$$= \left(\frac{2\omega_h}{\sigma_o}\right) \cos\left(\frac{\pi}{2x}\right) \Big/ \cos\left(\frac{\pi}{2x} n\right)$$

Synthesis of Equal-Q Maximally-Flat Prototype

$$|S_{11}(\omega)|^2 = \left(\frac{\omega^2}{\omega^2 + \sigma_o^2}\right)^n$$

where: $\sigma_o = 2\,\omega_h / q = 2\,g / c$

$$s_r = -\sigma_o$$

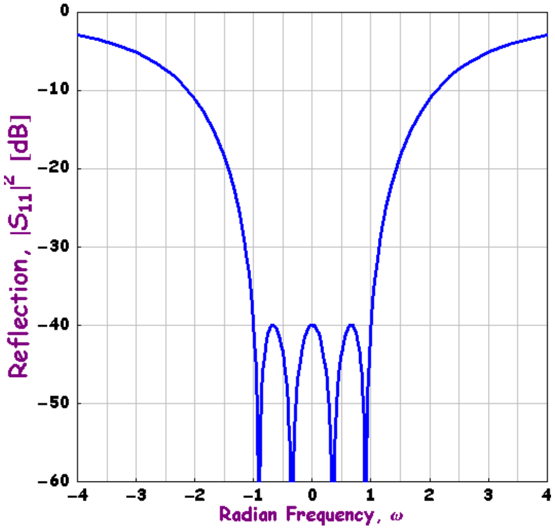
with admittance inverter values:

$$J_0 = \pm\sqrt{n\,g\,Y_s}$$

$$J_r = \pm g\sqrt{\frac{(n-r)(n+r)}{(2r-1)(2r+1)}} \quad \textbf{for} \quad r = 1\ldots(n-1)$$

For poles on the unit circle, choose $\sigma_o = 1$ **or** $q = \omega_h$
For 3-dB return loss at ω_h=1, choose $\sigma_o = (2^{1/n} - 1)^{1/2}$ **or** $q = 2\,\omega_h/\sigma_o = \omega_h c/g = 2\Big/\sqrt{2^{1/n} - 1}$
For return loss L_h at ω_h=1, choose $\sigma_o = (10^{L_h/(10n)} - 1)^{1/2}$ **or** $q = 2\,\omega_h/\sigma_o = \omega_h c/g = 2\Big/\sqrt{10^{L_h/(10n)} - 1}$

Lossy Equiripple Reflection Prototype

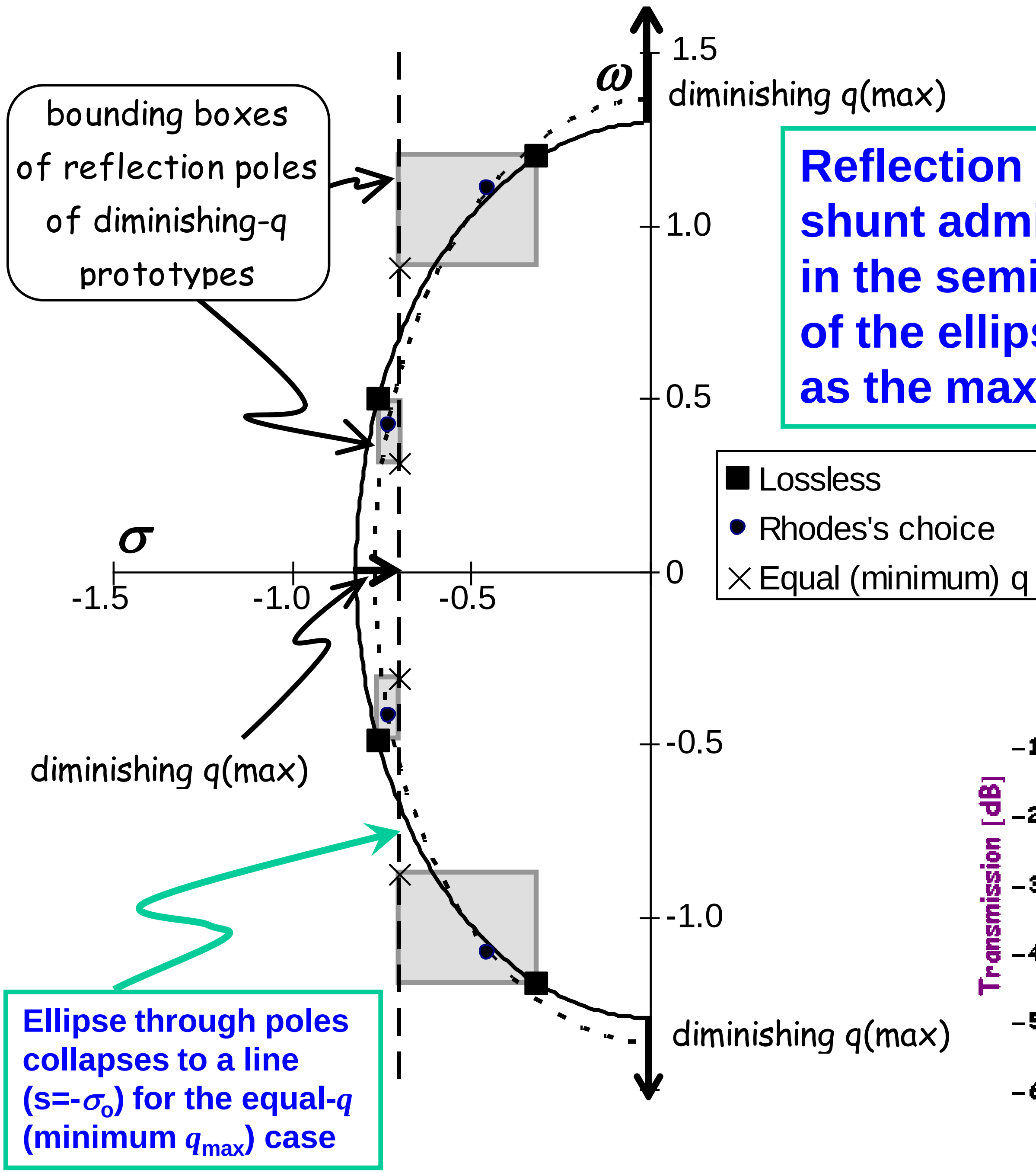


- **No generic equiripple reflection characteristic is known**
- **Analytic synthesis method for quasi-equiripple lossy reflection prototypes is known [5]**
- **Look-up table synthesis method for equiripple equal-q lossy reflection prototypes is now possible**

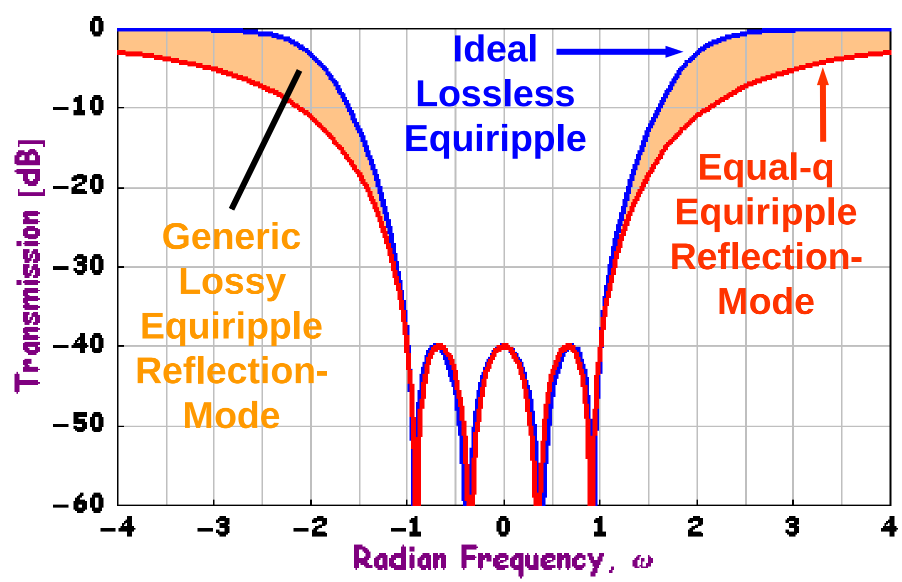
reflection characteristic	zeros	poles	θ_r	auxiliary parameters
equiripple lossless	$\cos\theta_r$	$\eta \sin\theta_r + j\sqrt{1+\eta^2}\cos\theta_r$	$\frac{\pi}{2}\left(\frac{2r-1}{n}\right)$	$\eta = \sinh\left(\frac{1}{n}\sinh^{-1}\sqrt{10^{L_S/10}-1}\right)$
quasi-equiripple lossy [5]	$\frac{\sigma_o}{\alpha}\cos\theta_r$	$\sigma_o\left(\sqrt{1-\alpha^{-2}}\sin\theta_r + j\cos\theta_r\right)$	$\frac{\pi}{2}\left(\frac{2r}{n+1}\right)$	$\alpha = 10^{-L_S/20}T_{n+1}(\alpha)$ $\sigma_o = \text{empirical}$
equiripple equal-q lossy	$\pm\omega_{zr}$	$-\sigma_o \pm j\omega_{zr}$	—	$\sigma_o = 2\omega_h/q,\ \omega_{zr} = \omega_h w_r$ Given n and L_S, look-up q and w_r in Table II.

Between extremes of infinite *q* and finite equal *q* is the continuum of possible equiripple characteristics for unequal-Q_u reflection filters

Lossy Equiripple Reflection Prototype



Reflection poles for 3 different sets of shunt admittance q, showing the trend in the semi-major and semi-minor axes of the ellipses through the poles as the maximum q diminishes



Synthesis of Lossy Equal-Q Equiripple Prototype

- **Look-up prototype order n and shunt admittance q_u for required equiripple stopband level L_S and maximum passband-to-stopband-edge frequency ratio**

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB
3	qu	1.89529	1.44730	1.13210	0.900258	0.724114	0.587102
	3.01 dB	2.32616	2.91915	3.63548	4.49696	5.53192	6.77583
	2 dB	2.81099	3.57034	4.48145	5.57178	6.87715	8.4423
	1 dB	3.90135	5.02447	6.36114	7.95196	9.84942	12.1188
	0.5 dB	5.45575	7.08219	9.00856	11.2941	14.0148	17.2644
4	qu	2.84833	2.23007	1.79171	1.46629	1.21650	1.01972
	3.01 dB	1.93142	2.32780	2.79069	3.32621	3.94202	4.64799
	2 dB	2.28375	2.79456	3.3859	4.06549	4.84313	5.73142
	1 dB	3.09075	3.85323	4.72606	5.72128	6.85375	8.14228
	0.5 dB	4.26008	5.37103	6.63382	8.06702	9.69281	11.5387

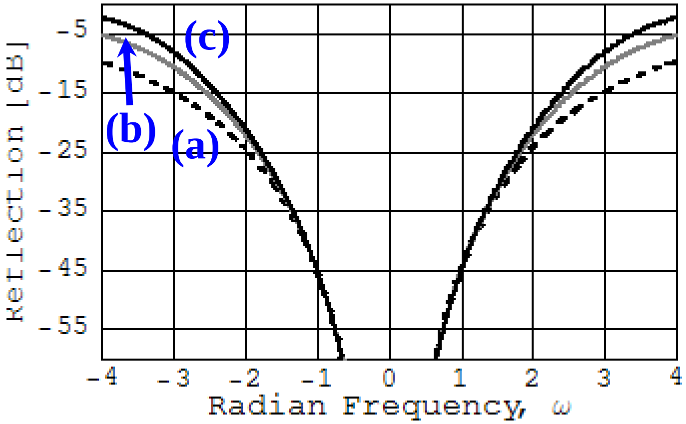
- **Look-up normalized prototype reflection zero frequencies for required equiripple stopband level L_S and order n ($\omega_{zr} = \omega_h W_r$)**

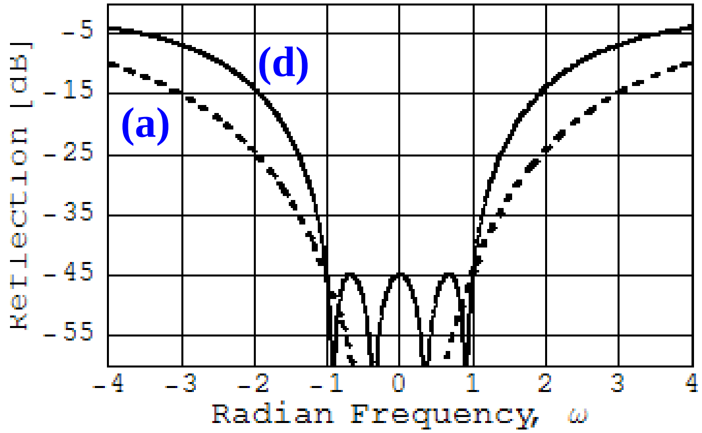
N	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB
3	qu	1.89529	1.44730	1.13210	0.90026	0.72411	0.58710
	$w2$	0.82011	0.83278	0.84225	0.84921	0.85424	0.85783
4	qu	2.84833	2.23007	1.79171	1.46629	1.21650	1.01972
	$w1$	0.31117	0.32120	0.33085	0.33983	0.34787	0.35484
	$w2$	0.87670	0.88730	0.89556	0.90198	0.90699	0.91091

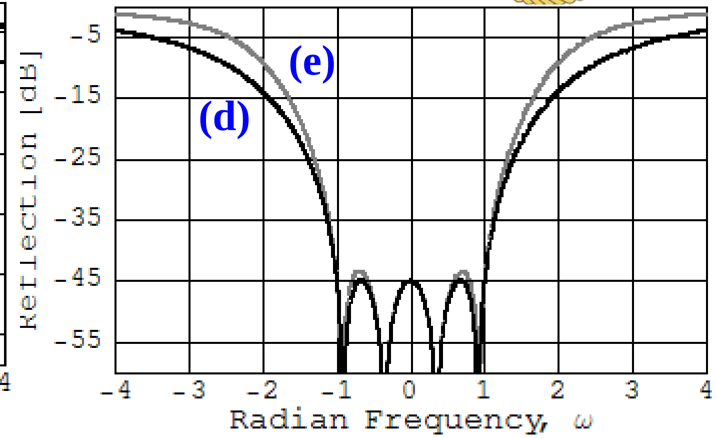
- **Look-up normalized prototype admittance inverter values for required equiripple stopband level L_S and for order n ($J_0 = \pm\sqrt{n\, g\, Y_S}$, $J_r = g\, \mathrm{J}_r$)**

n	L_S	20 dB	25 dB	30 dB	35 dB	40 dB	45 dB
3	q	1.89529	1.44730	1.13210	0.90026	0.72411	0.58710
	J1	2.06817	1.90660	1.80908	1.74823	1.70931	1.68397
	J2	1.06708	0.90420	0.79774	0.72674	0.67888	0.64644
4	q	2.84833	2.23007	1.79171	1.46629	1.21650	1.01972
	J1	2.91730	2.68593	2.54225	2.44923	2.38710	2.34454
	J2	1.72990	1.47760	1.30980	1.19442	1.11329	1.05528
	J3	1.23210	1.01535	0.86455	0.75636	0.67737	0.61908

Examples for $\{n=4,\ L_h=45,\ \omega_h=1,\ Y_s=0.02\}$







Example		q_1	q_2	q_3	q_4	L_0	k_1	k_2	k_3
Max-Flat, Equal-q, minimum q(max)	(a)	0.569	0.569	0.569	0.569	4.437	3.927	1.571	0.785
Max-Flat, Moderate q(max)	(b)	1.702	1.377	0.851	0.325	1.841	3.419	1.857	1.710
Max-Flat, Large q(max)	(c)	10.00	7.249	3.260	0.229	0.354	3.136	1.961	2.732
Equiripple, Equal-q, minimum q(max)	(d)	1.020	1.020	1.020	1.020	4.437	2.299	1.035	0.607
Quasi-Equiripple, Moderate q(max)	(e)	3.112	2.518	1.556	0.594	1.841	1.977	1.161	1.068

L_o = return loss due to coupling to first shunt admittance alone [dB] = $20\log\left|\dfrac{Y_s - J_0^2/g_1}{Y_s + J_0^2/g_1}\right|$

k_r = coupling between the r^th and (r+1)^th prototype shunt admittances = $\dfrac{J_r}{\sqrt{c_r\ c_{r+1}}}$

Note: The r^th admittance inverter value J_r can be modified by scaling the admittance matrix's (r+1)^th row and column by a constant [11], which will also scale shunt c and g values, but will leave the reflection characteristic and shunt admittance q unchanged.

Conclusion

- Lossy reflection-mode bandstop filters have
  - "ideal" stopband characteristics
  - "lossy" passband characteristics
  - better passband loss than predistorted filters
  - selectivity proportional to $Q_{u,max}$
- Analytic synthesis of generic lossy maximally-flat bandstop filters is now possible
- Look-up-table synthesis of equal-Q equiripple-stopband bandstop filters is now possible
- Bounds for poles and zeros of generic lossy equiripple bandstop filters have been quantified

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